THE UNIVERSITY OF ALBERTA

NONLINEAR DISCRETE FEEDFORWARD CONTROLLERS

by

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A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled 'Nonlinear Discrete Feedforward Controllers' submitted by G. Richard Kampjes in partial fulfilment of the requirements for the degree of Master of Science.

Date 28

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ABSTRACT

In the synthesis of sampled-data control systems, the criteria of performance are expressed in the form of time domain specifications, such as overshoot, peak time and damping ratio. Because of the flexibility in the application and realizability of a digital controller, a sampled-data system can be designed to exhibit deadbeat response whenever a specific test signal such as a step, ramp or parabolic input is applied.

If however, the system contains nonlinear elements, the design of the digital controller depends not only upon the type of test signal applied but upon the magnitude as well.

In this thesis a 'feedforward' method is used to compensate for the nonlinearity, so that the system exhibits deadbeat response regardless of the magnitude of the step input.

Most references on the subject consider systems in which the nonlinear element can be isolated from the remainder of the plant. In most practical situations this is not the case and the nonlinearity is in fact an integral part of the plant. This thesis considers both cases and develops two methods for determining the type of feedforward compensation required in each case.

Although perfect deadbeat response was not obtained for large signals, a marked improvement was obtained over the case in which no compensation was used.

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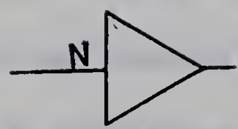
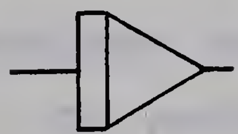
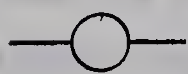
AND DEFINITIONS



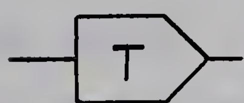
Amplifier (Gain = N)



Integrator

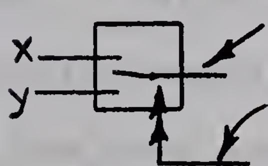


Potentiometer



Unit Time Delay

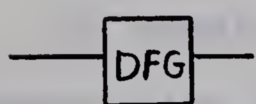
Multiplier



Armature

Comparator (armature to x for pos. v)
(armature to y for neg. v)

Controlling
Voltage = v



Diode Function Generator

Manual Switch

$$\% \text{ Overshoot} = \frac{(\text{max. neg. error after time } 2T)\ 100\%}{R_o}$$

$$\% \text{ Undershoot} = \frac{(\text{max. pos. error after time } 2T)\ 100\%}{R_o}$$

CHAPTER ONE

INTRODUCTION

In the study and design of feedback control systems, performance specifications cannot usually be met merely by adjusting certain parameters of the existing system. The study of compensation techniques then becomes of great importance.

The problem which is considered in this thesis is that of compensating a nonlinear sampled-data system so that it exhibits deadbeat response for a step input. A linear system can be compensated so that the output exhibits minimal (or deadbeat) response regardless of the magnitude of the step input. For a nonlinear system, however, the compensation required is dependent upon the magnitude of the step input.

In sampled-data systems, the principle of compensation is similar to that for continuous-data systems. The output response in both cases is characterized by the usual terms of specifications such as per-cent overshoot, rise time, steady state error and so forth. Because of the sampling operation, the design of compensating devices for sampled-data systems is usually more complicated than for continuous-data systems. However, because

of sampling, the design procedure is usually much more versatile.

The method of compensation which will be considered in this thesis is accomplished by introducing digital controllers into the system. A digital controller is simply a pulsed-data network, of which the transfer function can be expressed by:

$$D(z) = \frac{a_o + a_1 z^{-1} + a_2 z^{-2} + \dots + a_m z^{-m}}{b_o + b_1 z^{-1} + b_2 z^{-2} + \dots + b_n z^{-n}}$$

where n is larger than or equal to m. The terms in the numerator represent the inputs at the various sampling instants. Similarily, the terms in the denominator represent the values of the output of the digital controller at the sampling instants. In general n is equal to m and since the input and output values are valid only at the specific sampling instants, the transfer function can be written as:

$$D(z) = K_o z^o + K_1 z^{-1} + \dots + K_n z^{-n} .$$

The digital controller can now simply be regarded as a variable gain element, in which the gain is constant throughout any sampling period.

In this thesis, the following general configuration shall be considered. (Fig. 1-1)

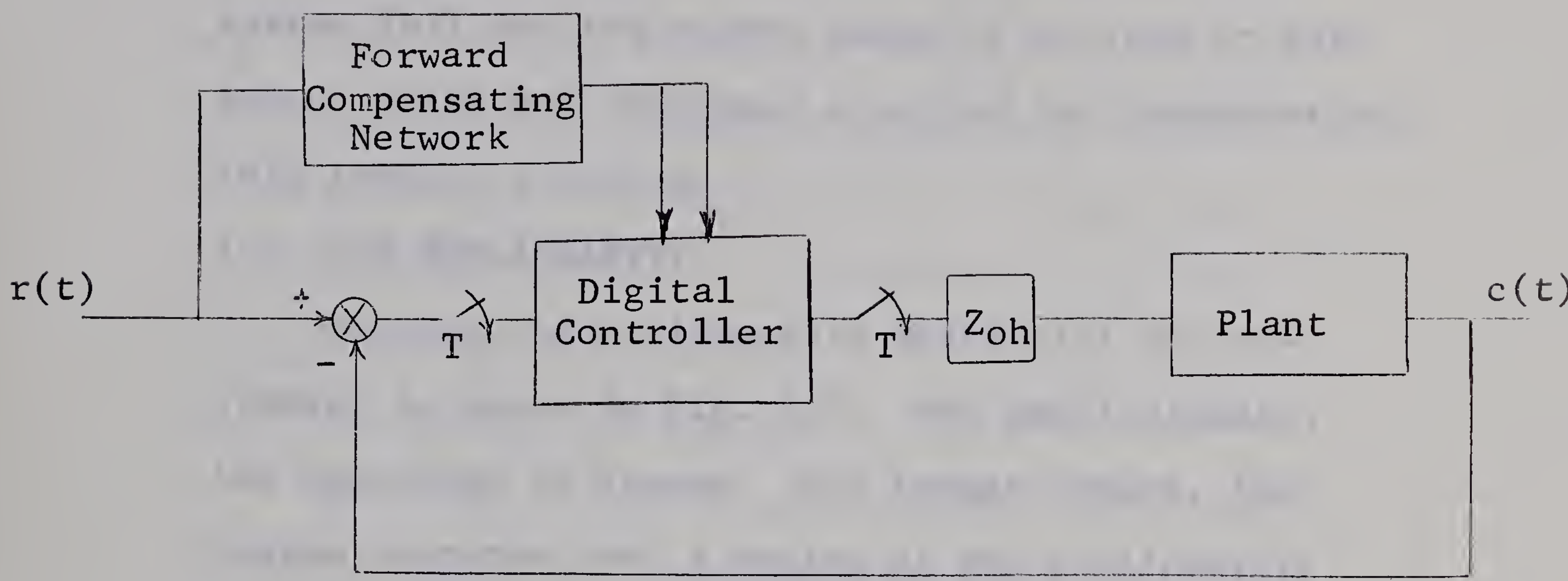


Fig. 1-1

The desired output will be deadbeat response and the only type of input signal which will be considered is a step input.

1-1 THE PLANT

The plant is a simple Type 1 system with a transfer function, $G(s) = 1/s(s+1)$. In addition, however, it will contain a saturation type of nonlinearity. Two situations will be considered. The first case, in which the nonlinearity is isolated from the plant, is the case which most authors consider in examples of nonlinear compensation. The approach is usually quite straight-forward. In the second case, the nonlinearity is considered to be "sandwiched" between the two elements of the plant.

This is the situation found most often in physical systems. All normal attempts at analysing this system fail and the author knows of no text or reference which has developed a method for compensating this type of a system.

1-2 THE NONLINEARITY

The type of nonlinearity which will be considered is shown in Fig. 1-2. For small signals, the operation is linear. For larger inputs, the system operates over a region of the nonlinearity where the effective gain is reduced. In this region the nonlinearity is assumed to be a straight line with slope K, where K is less than 1.

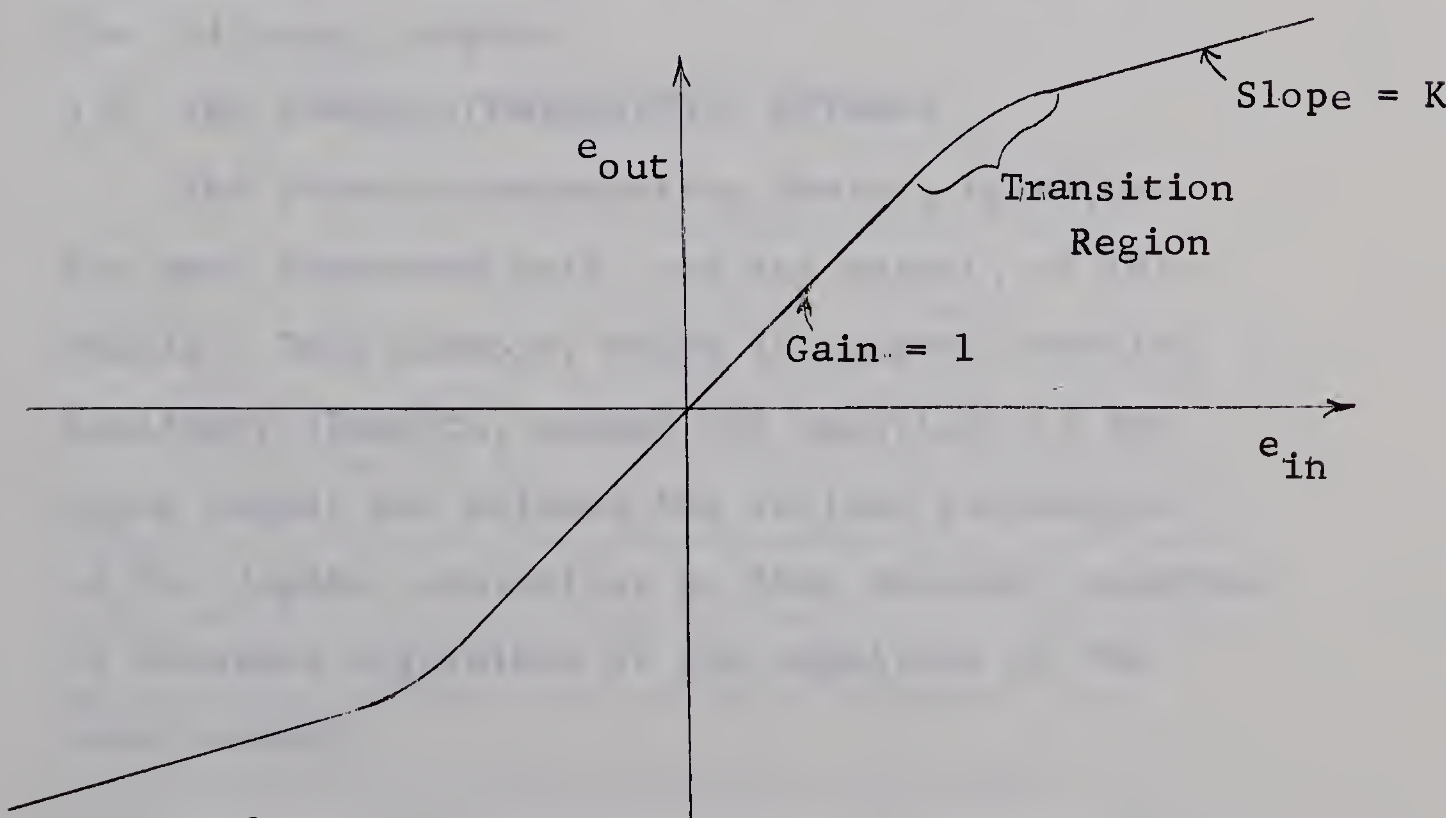


Fig. 1-2

The gradual transition between the two segments was included to cover the general case in which the transition may not be so abrupt. The methods developed in this thesis are completely general in that the gradual transition can be any shape and the slope, K, can be varied from 0 to 1 and the process is still valid.

1-3 THE DIGITAL CONTROLLER

The digital controller, D(z), is the compensating, pulsed-data network which would normally be used with a linear system to obtain deadbeat response. The design of and the procedure for simulating the controller will be outlined in the following chapter.

1-4 THE FORWARD-COMPENSATING NETWORK

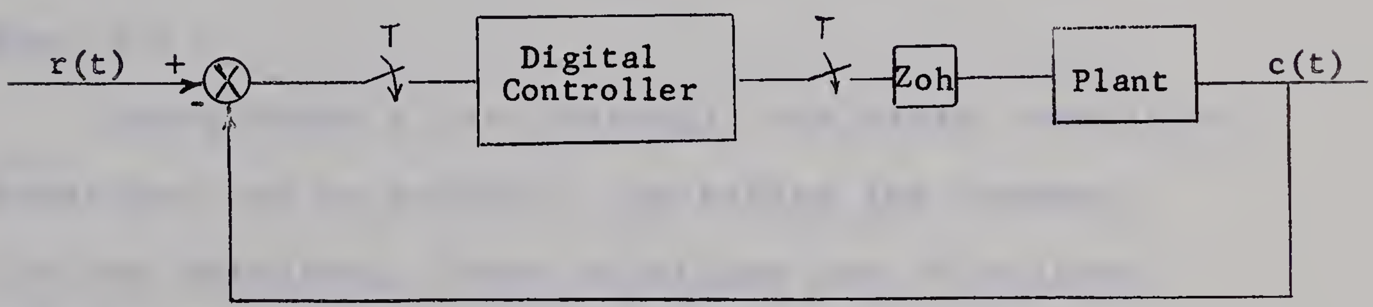
The forward-compensating device, $D_f(z)$, is the most important part, and the object, of this thesis. This network, which in general contains nonlinear elements, senses the magnitude of the input signal and adjusts the various parameters of the digital controller so that deadbeat response is obtained regardless of the magnitude of the input signal.

CHAPTER 2

THE LINEAR SYSTEM

In the design of the digital controller for the linear system, the state transition approach shall be used. Using this approach, the state transition equations are used to describe the state of each variable at any given time. This method insures that there will be no inter-sampling ripple and that the response is indeed deadbeat.

The closed-loop, linear system is shown below in Fig. 2-1.



Sampling period = T

$G(s) = 1 / s(s+1)$

Fig. 2-1

$D(z) = K_o + K_1 z^{-1} + \ldots + K_n z^{-n}$

When drawing the transition flow graph, the digital controller can be considered as a variable gain network as discussed in Chapter 1. The zero-order hold measures the output of D(z) at the beginning of the sampling period and holds this value through one sampling period. The transition flow graph is shown in Fig. 2-2.

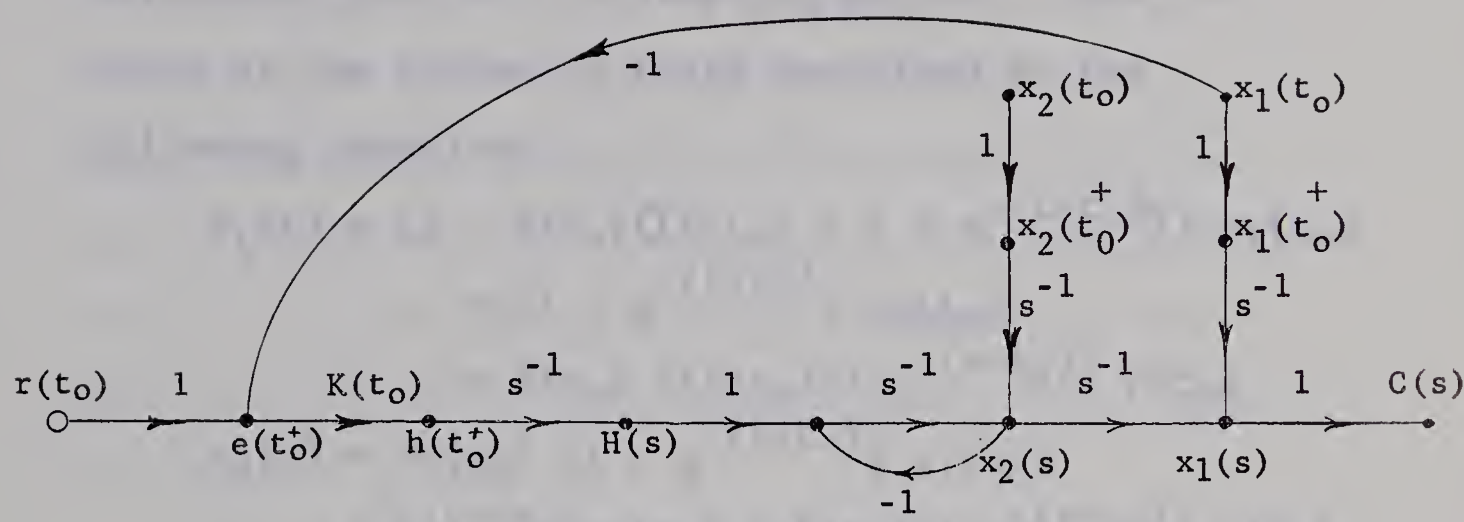


Fig. 2-2

Using Mason's Gain Formula, the state transition equations can be written. By taking the inverse Laplace transform, these equations can be written in the time domain. This set of independent equations describes the response of the state variables during a certain period of time. For the general system, having n state variables, n conditions must be satisfied by n equations. The only unknowns in the system are K_o, K_1, ... , K_{n-1}. Thus n unknowns are required to solve the n simultaneous equations. This implies that n sampling periods are required to obtain a specified response from an 'n'th order system. Thus, in the above example, the minimum time required to obtain

deadbeat response is two sampling periods. The two equations are continuous for a specified 'n', throughout the 'n+1' st sampling period. Thus the state of the system is fully described by the following equations.

$$x_1(t) = (1 - K(t_o)((t-t_o) - 1 + e^{-(t-t_o)})) \, x_1(t_o) + (1 - e^{-(t-t_o)}) \, x_2(t_o) + K(t_o) \, ((t-t_o)-1+e^{-(t-t_o)}) \, r(t_o)$$

$$x_2(t) = -K(t_o) \, (1 - e^{-(t-t_o)}) \, x_1(t_o) + e^{-(t-t_o)} x_2(t_o) + K(t_o)(1-e^{-(t-t_o)}) \, r(t_o)$$

For $t_o = kT$, $t = (k+1)T$, $r(t) = R_o$ and K_k = the gain of D(z) during the 'k+1'th sampling period;

$$x_1((k+1)T) = (1 - K_k(T-1+e^{-T})) \, x_1(kT) + (1-e^{-T}) \, x_2(kT) + K_k(T-1+e^{-T}) \, R_o \tag{2-1}$$

$$x_2((k+1)T) = -K_k(1-e^{-T}) \, x_1(kT) + e^{-T} x_2(kT) + K_k(1-e^{-T}) \, R_o \tag{2-2}$$

The conditions for deadbeat response to a step input of magnitude R_o are:

(a) The output, $x_1(t)$, must equal R_o.

(b) All other state variables must be zero.

It is required that the system approach these conditions within as few sampling periods as possible. Thus, for k = 0, with all initial conditions being zero, it is required that:

$$x_1(T) = K_o(T - 1 + e^{-T})\, R_o = R_o$$

$$x_2(T) = K_o\,(1 - e^{-T})\, R_o = 0$$

From these equations, it is verified that it is not possible to obtain deadbeat response in one sampling period. Assuming that deadbeat response can be obtained in two sampling periods, the above values of $x_1(T)$ and $x_2(T)$ can be substituted into equations 2-1 and 2-2 as initial conditions. Then letting $k = 1$:

$$x_1(2T) = (1 - K_1(T-1+e^{-T}))K_o(T-1+e^{-T})\, R_o$$
$$+ (1 - e^{-T})\, K_o\,(1 - e^{-T})\, R_o$$
$$+ K_1(T - 1 + e^{-T})\, R_o = R_o \quad \text{and}$$

$$x_2(2T) = -K_1(1 - e^{-T})\, K_o\,(T - 1 + e^{-T})\, R_o$$
$$+ e^{-T}(1 - e^{-T})\, K_o\, R_o$$
$$+ K_1\,(1 - e^{-T})\, R_o = 0.$$

Solving these equations simultaneously yields:

$$K_o = 1\,/\,T(1-e^{-T})$$

$$\text{and} \quad K_1 = 1\,/(1+T-e^{+T})$$

If a sampling period of one second is used, the values of K_o and K_1 are:

$$K_o = 1.5820$$

$$\text{and} \quad K_1 = -1.3922$$

$$x_1(T) = R_o/\,(e-1) = .5820\, R_o$$

$$e(T) = R_o - x_1(T) = .4180\, R_o$$

$$h(T^+) = K_1\, e(T) = .4180\,(-1.3922)R_o = -.5820\, R_o$$

Thus: $$D(z) = \frac{1.5820 - .5820\, z^{-1}}{1 + .4180\, z^{-1}}$$

2-1 SIMULATION

Once D(z) has been calculated, the system can be simulated on the analog computer. To simulate the digital controller, the circuit shown in Fig. 2-3 is most practical. The design and operation of the controller is described in Appendix A. With reference to Fig. 2-3, the initial error signal, $e(0^+)$, which is equal to R_o, is multiplied by K_o and held by the zero-order hold for one sampling period. At the end of one sampling period, the output of the plant, $x_1(t)$, will have reached a value of .5820 R_o. Thus e(T) is equal to .4180 R_o. Since the potentiometer, P_2, is set at a value of .4180, then the output of amplifier A will be zero at the beginning of the second sampling period.

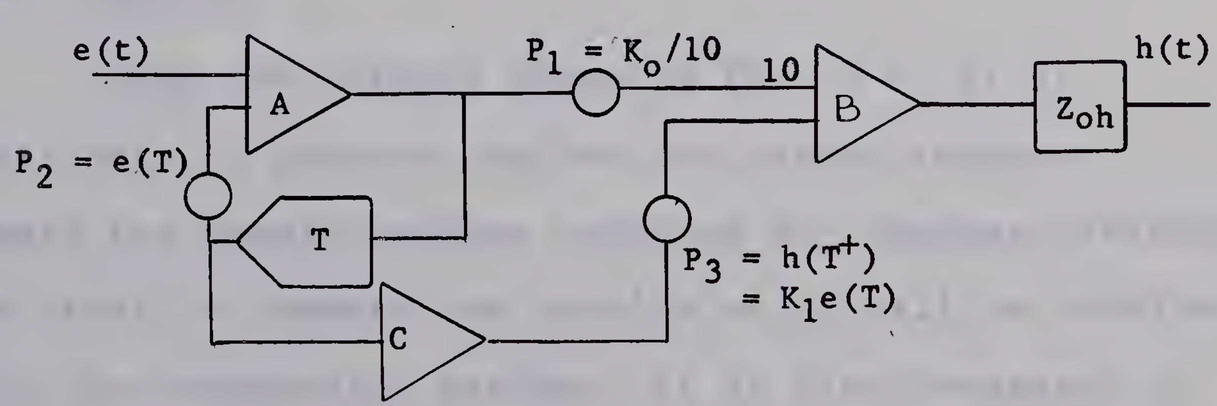


Fig. 2-3

Since the potentiometer, P_3, is set at .5280, the output of amplifier B will be equal to -.5280 R_o at the instant T^+. The zero-order hold then holds this value for one sampling period and the system responds to this negative step. At the end of the second sampling period, the output of the system will be equal to R_o and the state variable, $x_2(t)$, will be equal to zero. Thus deadbeat response has been achieved. It should be noted that:

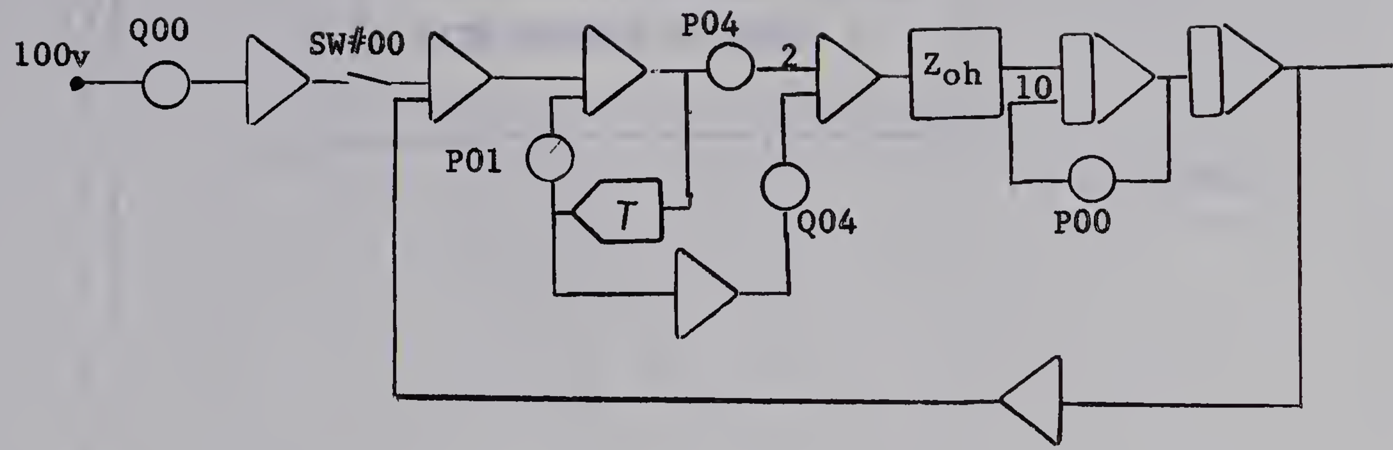
(a) since the output of Amplifier A is zero at the instant, T^+, the output from the time delay will be zero for all time after 2T.

(b) since the output of the plant is equal to R_o at t = 2T, the error signal will be zero for all time after 2T.

The complete computer schematic for this linear system is shown in Fig. 2-4.

2-2 RESULTS

Using the network shown in Fig. 2-4, it is desirable to observe whether the system response meets the specifications required for deadbeat response. In order to compare the results which will be obtained for the compensated systems, it is also necessary to observe some measure of sensitivity for this linear system. To do this, certain parameters can be adjusted.

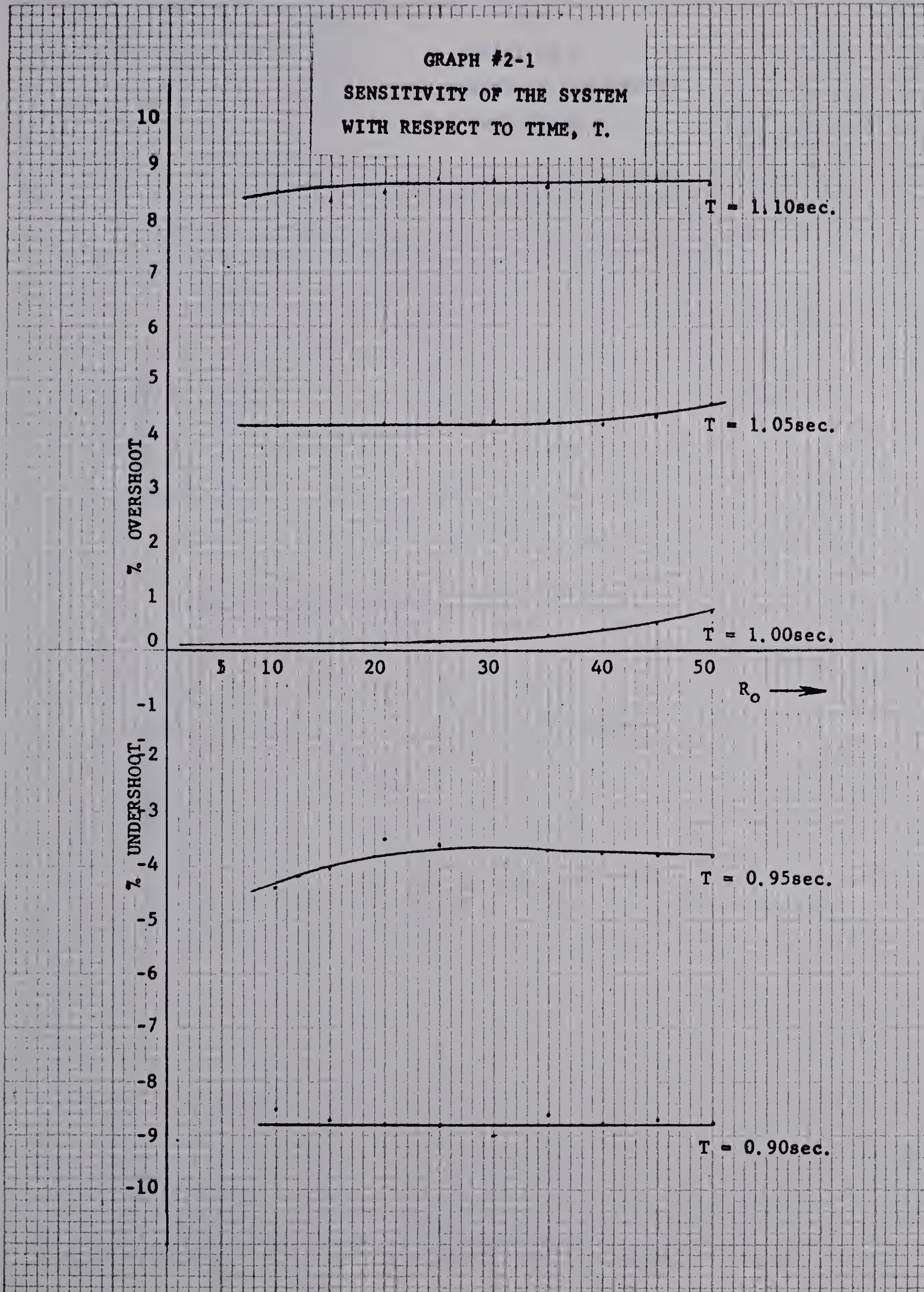


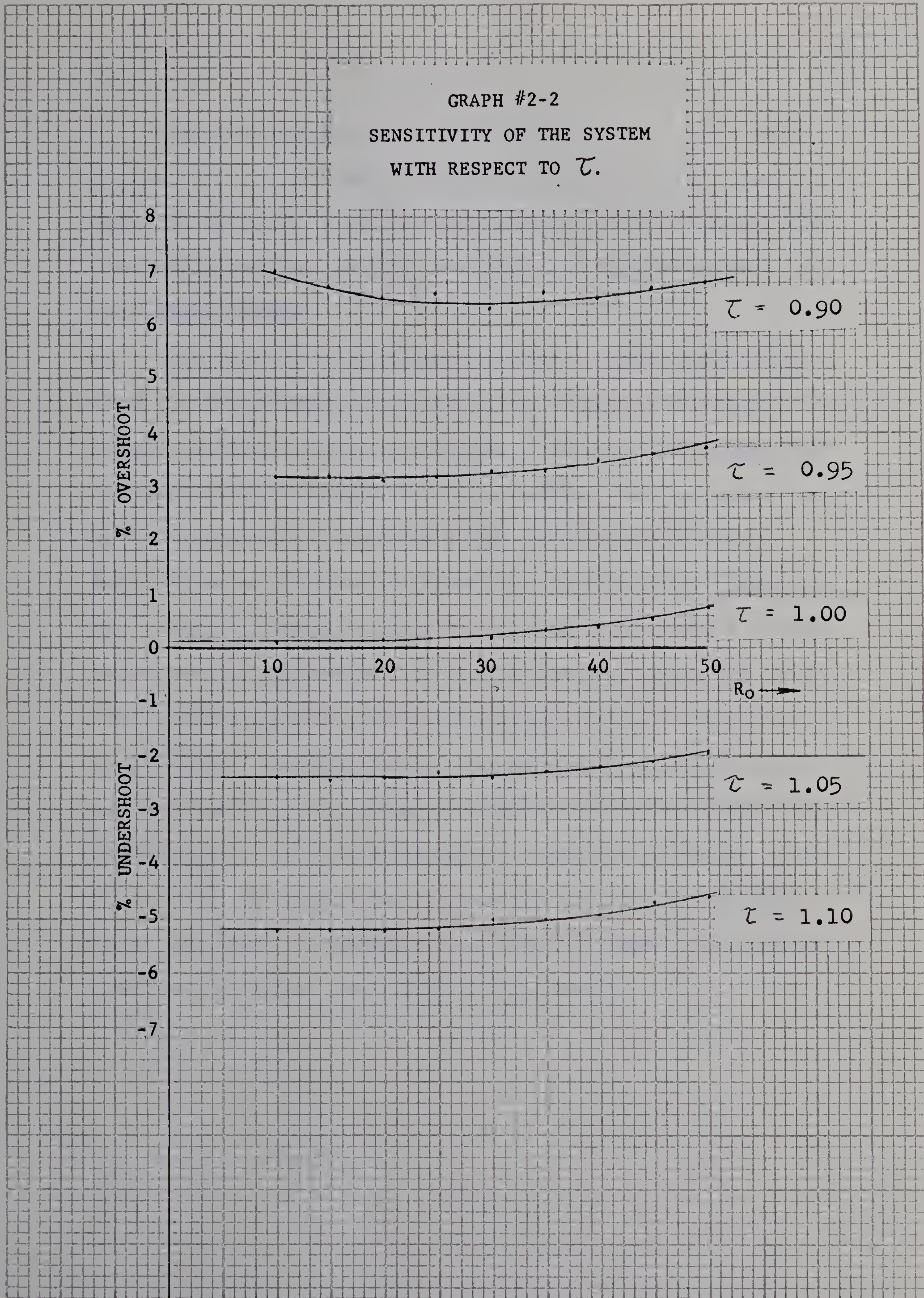
P00 = .100
Q00 = $R_o/100$
P01 = .4180
P04 = $K_o/2$ = .7910
Q04 = .5820

Fig. 2-4

The percentage deviation from the desired response can then be measured. If, for example, in the actual plant, the sampling period, T, and the time constant, τ, are not known to any degree of accuracy, then the digital controller will not yield perfect deadbeat response. The percentage overshoot (or undershoot) for various changes in T and τ are shown in graphs 2-1 and 2-2.

The results indicate that if the sampling period is varied by 10% from the value for which it was designed, the response would be in error by no more than 9%. There is an almost linear relationship between the percentage change in the sampling period and the percentage overshoot of the response.


GRAPH #2-1
SENSITIVITY OF THE SYSTEM
WITH RESPECT TO TIME, T.
% OVERSHOOT
% UNDERSHOOT
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
5
10
20
30
40
50
R_o
T = 1.10sec.
T = 1.05sec.
T = 1.00sec.
T = 0.95sec.
T = 0.90sec.


GRAPH #2-2
SENSITIVITY OF THE SYSTEM
WITH RESPECT TO τ.
% OVERSHOOT
% UNDERSHOOT
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
10
20
30
40
50
R_o
τ = 0.90
τ = 0.95
τ = 1.00
τ = 1.05
τ = 1.10

Similar results occur for variations in the time constant, τ. The sensitivity is almost constant at 0.5, meaning that for a percentage change of 10% in the time constant, the percentage overshoot is approximately 5%.

2-3 CONCLUSIONS

The results illustrate how the digital controller, used in the linear system, can be designed to obtain minimal response to an accuracy of less than 1%. Although this system is not insensitive to parameter variations, a certain amount of variation can be allowed and the deterioration in the response is not drastic. Thus if, for example, the designer is willing to tolerate a 10% overshoot or undershoot in the response, then the sampling period can be allowed to drift by as much as 10%. In practical applications, the sampling period can usually be kept accurate to within 0.1 to 0.01 percent. Similarily, the time constant of the system needs to be known only to within approximately 10% of its actual value in order to design a controller which will yield deadbeat response to within 5% accuracy.

CHAPTER 3

THE NONLINEAR SYSTEM WITH ISOLATED NONLINEARITY

The nonlinearity which was considered cannot, in general, be described exactly by any mathematical expression. The analysis of the system response for various magnitudes of inputs cannot be done by using strictly mathematical approaches. An attempt was made to approximate the curve (shown in Fig. 3-1) using an expression of the form:

$$e_o = (e_i + aKe_i^2) \;/\; (1 + ae_i) \;,$$

and use a complimentary function in the compensating network. This approach was abandoned for the following reasons.

(a) The difference between the actual curve and the curve described by the approximate function could, in some cases, make the use of this method impractical. Because of this there is a loss in generality.

(b) If the actual curve did indeed correspond exactly to the approximation, the error introduced in compensating the system with the complimentary mathematical expression was in some cases too large.

3-1 COMPENSATION

To compensate for the nonlinearity, consider the system shown in Fig. 3-2.

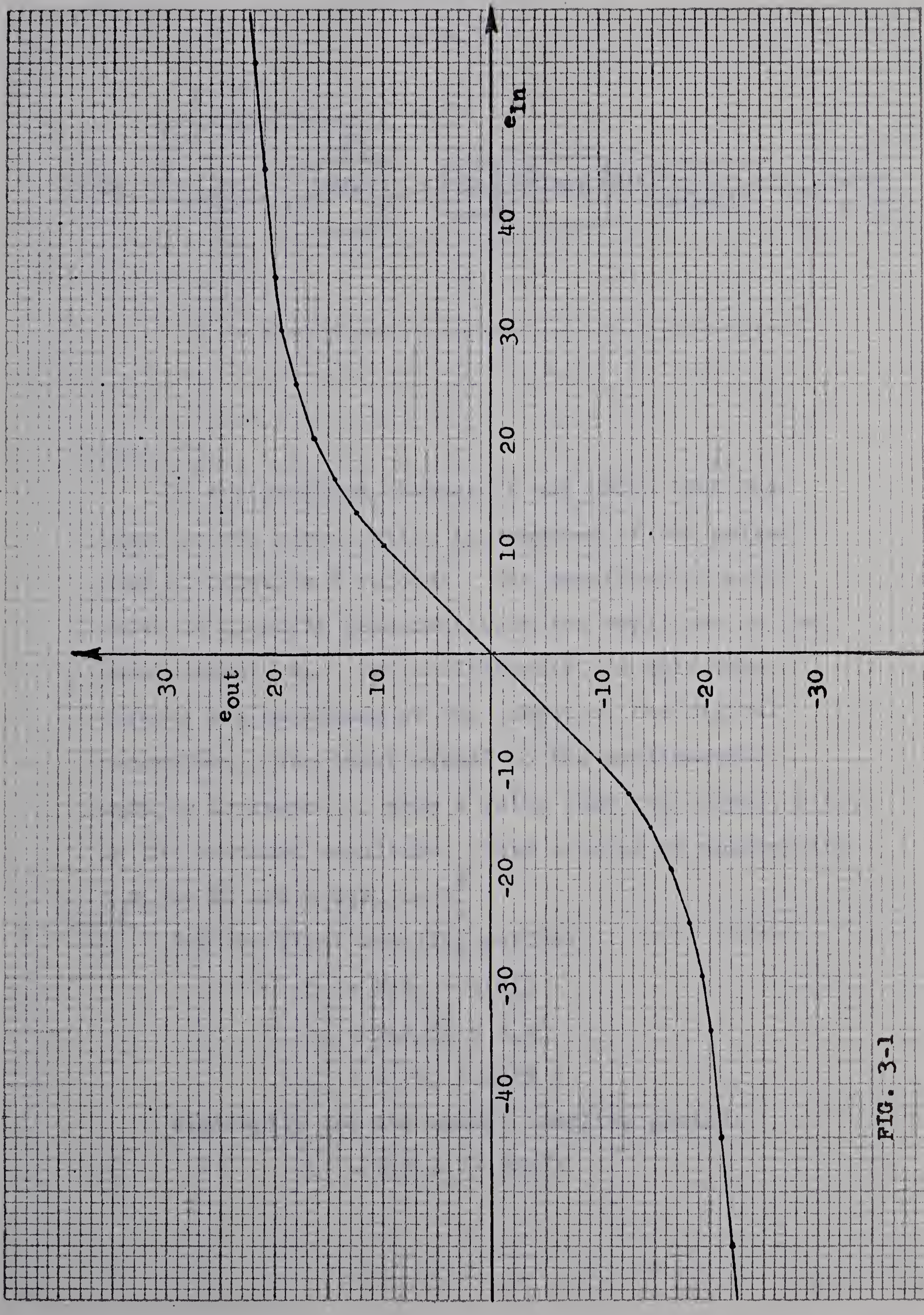
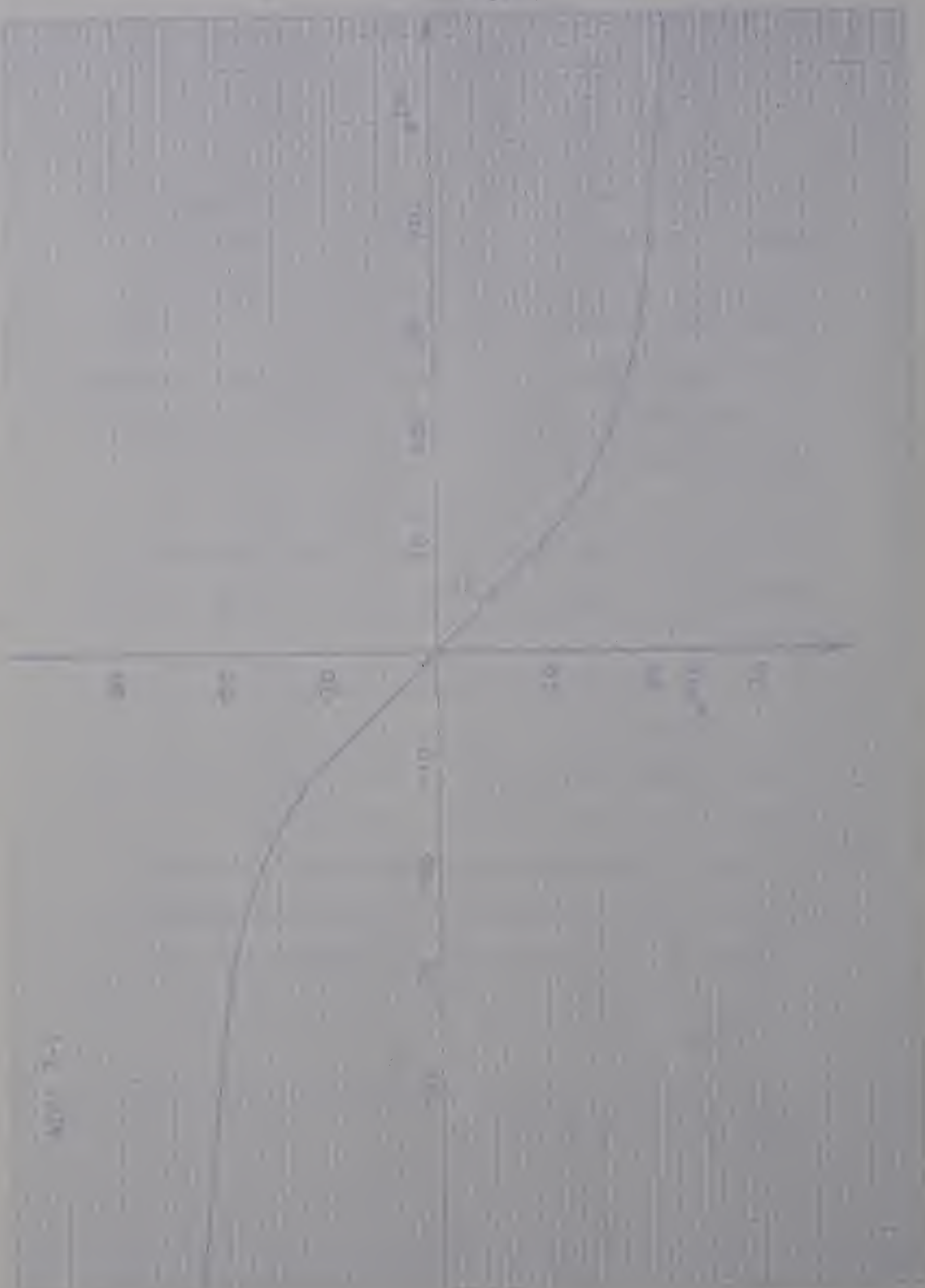


FIG. 3-1



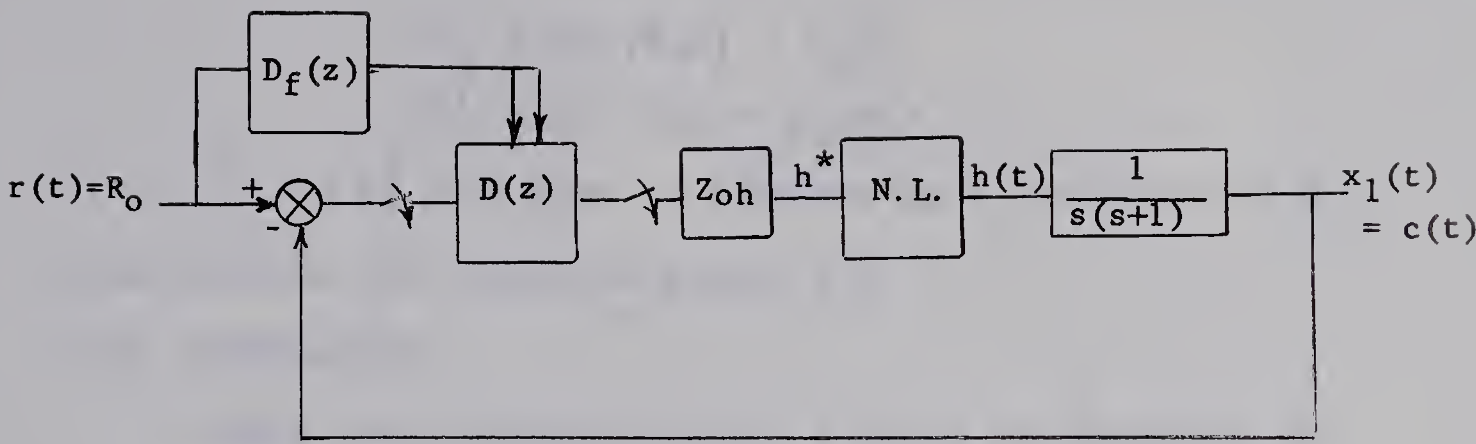


Fig. 3-2

In the previous chapter it was shown that the input to the plant, h(t), is composed of two pulses, each of duration T seconds. The magnitude of each pulse is linearly dependent upon the amplitude of the input signal, R_o. The nonlinearity, in this case, affects the magnitude of the output of the digital controller. The input signal to the nonlinearity must be increased to such a value that the output, h(t), is the required magnitude. This is done by multiplying K_oR_o by K_o^f and $e_1K_1R_o$ by K_1^f.

For the first sampling period:

$$h_o = K_oR_o = f(h_o^*)$$
$$h_o^* = K_oR_oK_o^f = h_oK_o^f$$
$$K_o^f = h_o^*/h_o = g_o(R_o)$$

Similarily for the second sampling period:

$$h_1 = R_oK_1e_1 = f(h_1^*)$$

$$h_1^* = K_1 e_1 R_o K_1^f = h_1 K_1^f$$

$$K_1^f = h_1^* \,/\, h_1 = g_1(R_o)$$

K_o^f and K_1^f can then be plotted as a function of R_o. The results are shown on graph 3-2.

3-2 SIMULATION

Using the results of Sec. 3-1, it is possible to simulate a system to test the validity of this feed-forward method of compensation. This system is shown in Fig. 3-3. A similar system, with no feedforward compensation, was simulated to illustrate the effectiveness of the compensation. The results are shown in Graph 3-1.

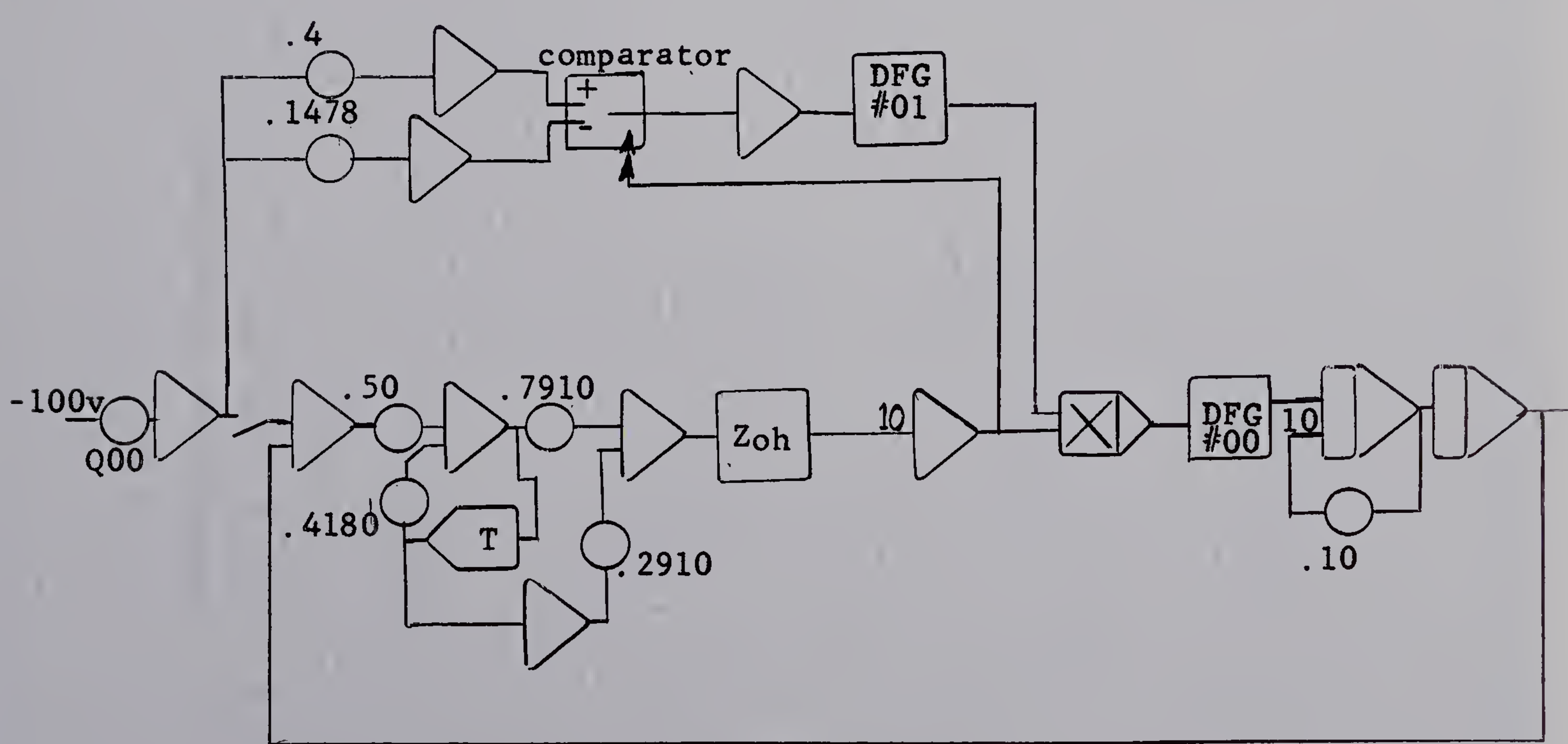
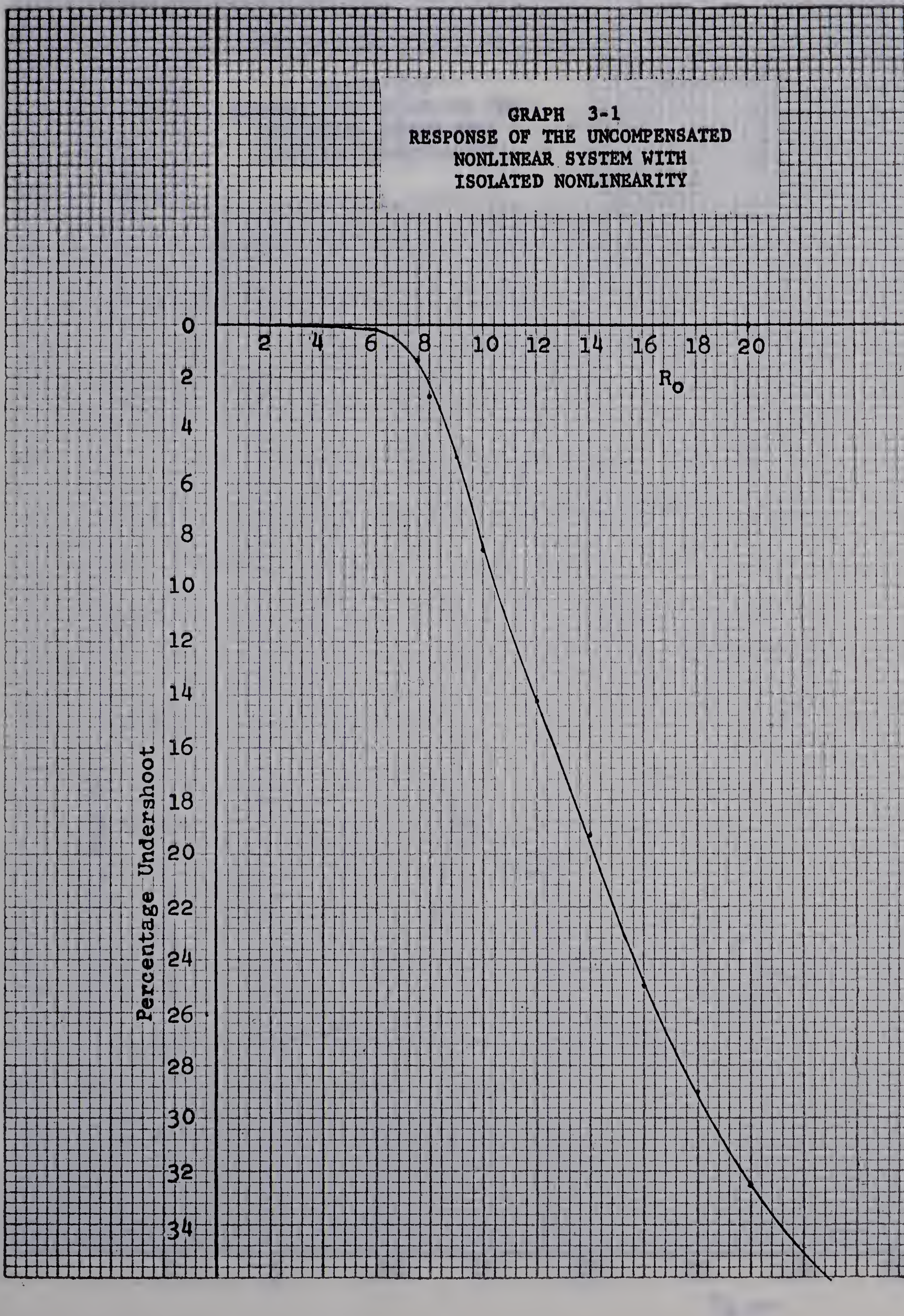
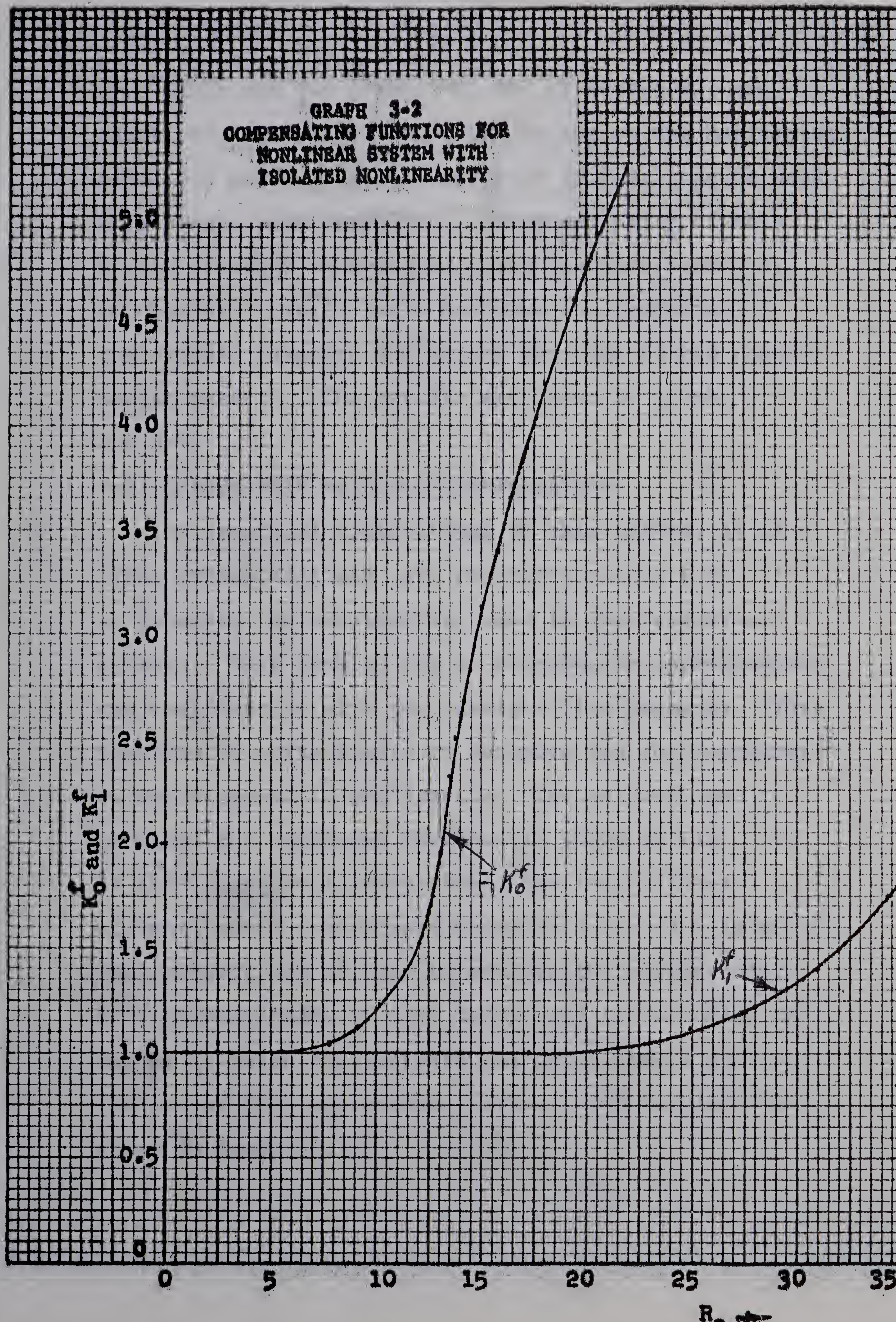


Fig. 3-3


GRAPH 3-1
RESPONSE OF THE UNCOMPENSATED
NONLINEAR SYSTEM WITH
ISOLATED NONLINEARITY
Percentage Undershoot
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
34
2
4
6
8
10
12
14
16
18
20
R0

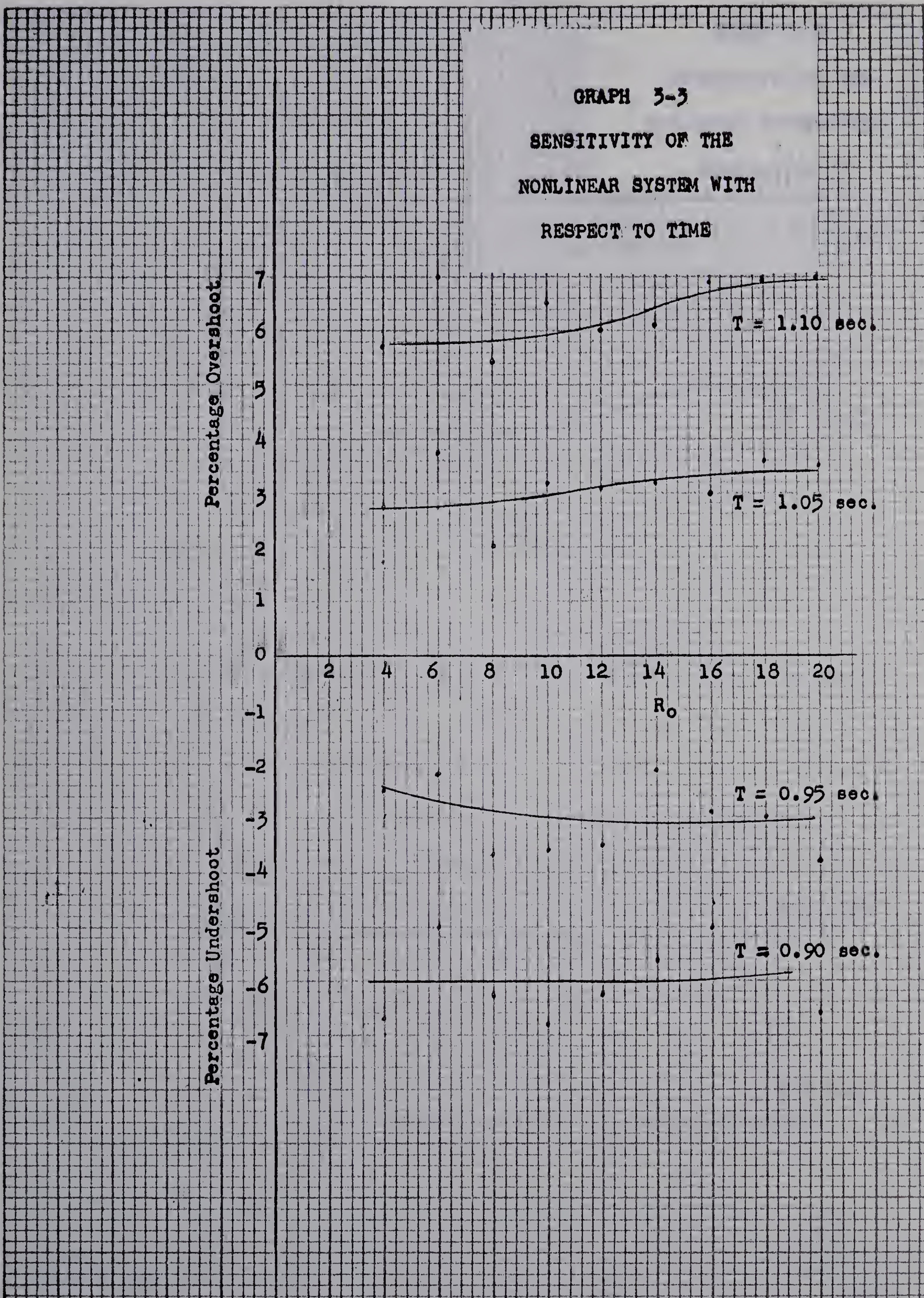


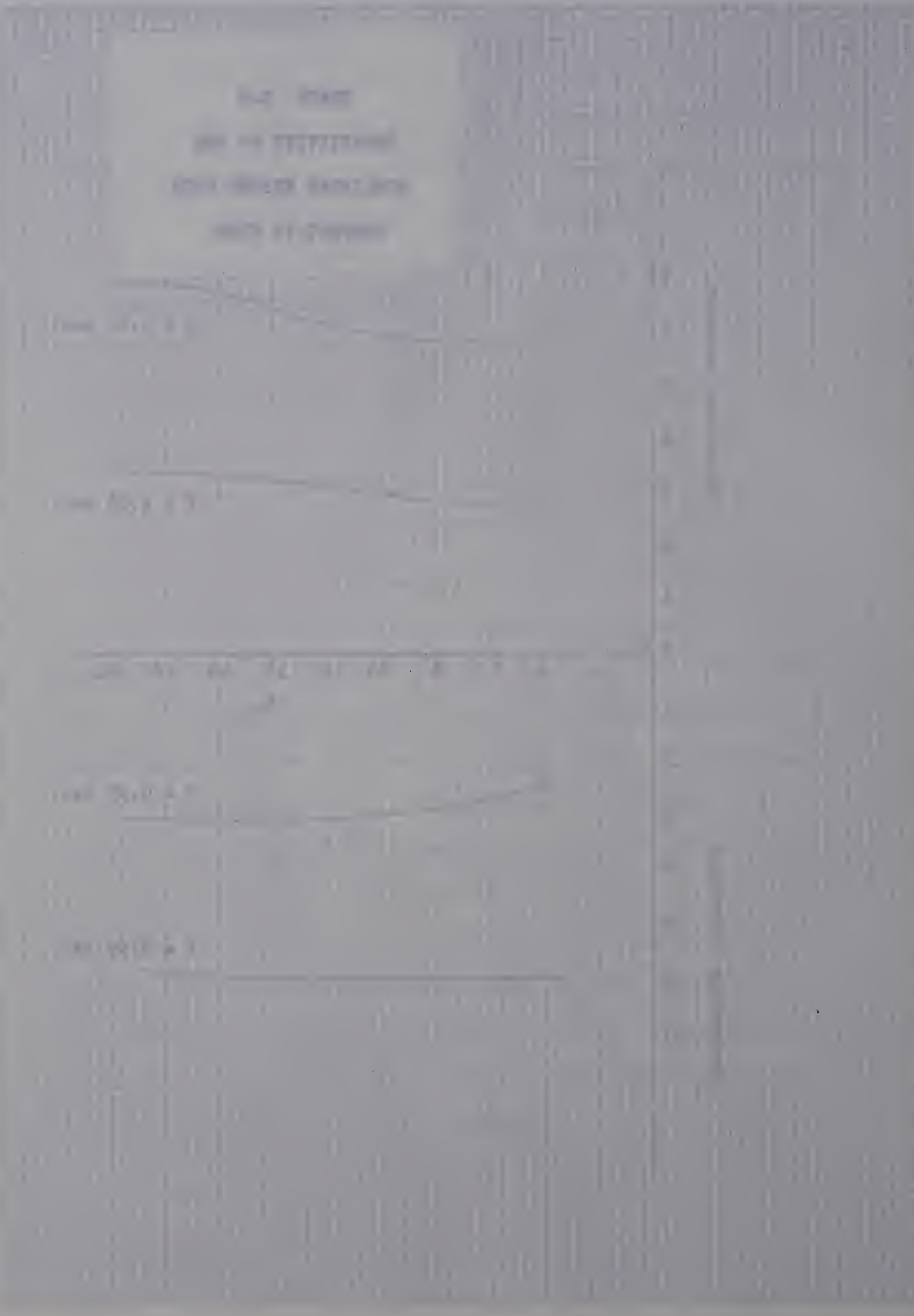
GRAPH 3-2
COMPENSATING FUNCTIONS FOR
NONLINEAR SYSTEM WITH
ISOLATED NONLINEARITY
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0
K_o^f and K_I^f
K_o^f
K_1^f
0
5
10
15
20
25
30
35
R_o

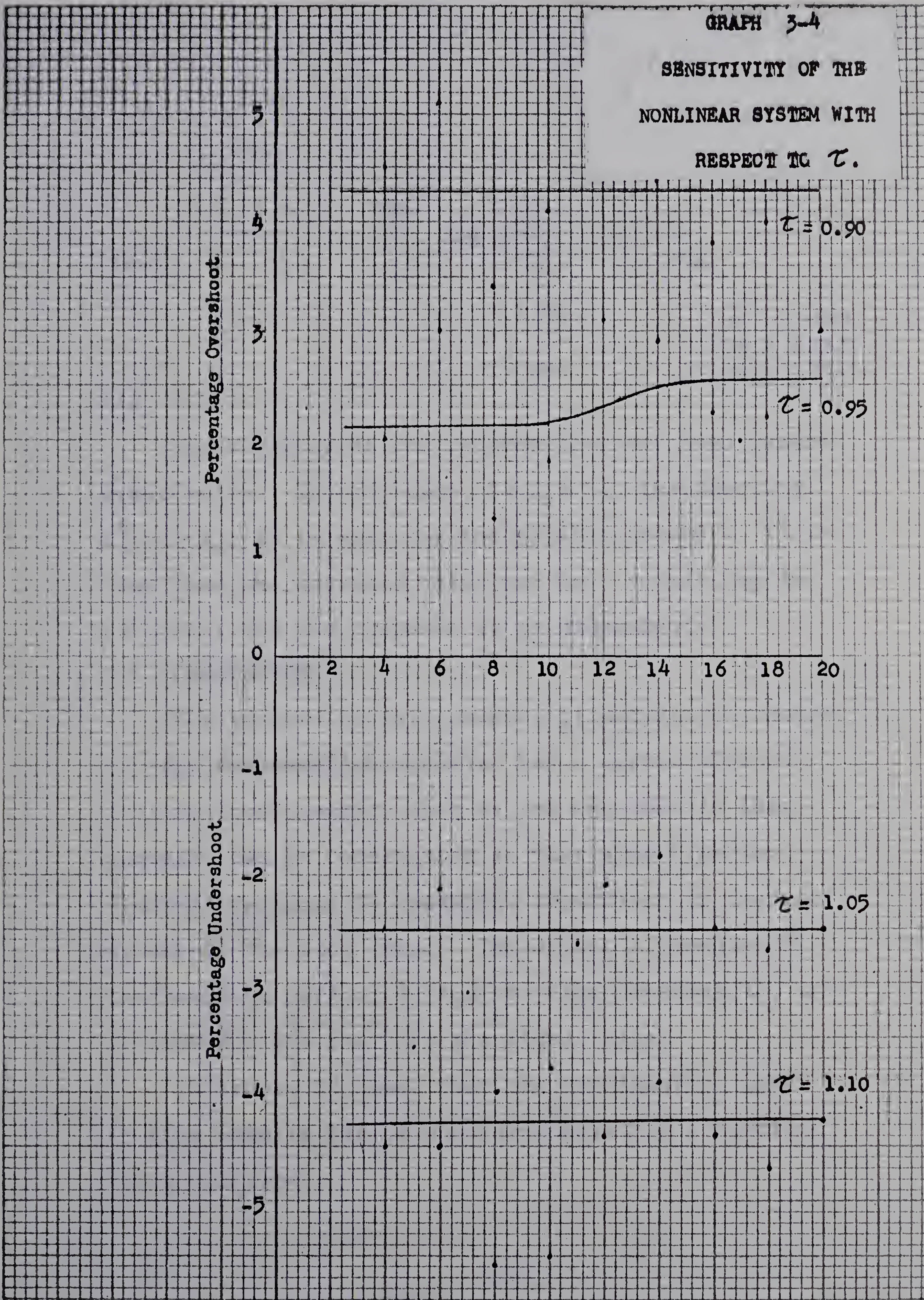
Since the two graphs, K_o^f and K_1^f versus R_o, are identical, except for the scale factor on the horizontal axis, one Diode Function Generator (D.F.G.) can be used to generate both functions. The sensitivity of this system can then be measured using the same method as in Chapter 2, to see if there has been any degrading of the overall response due to the addition of the compensating network. The results are shown on graphs 3-3 and 3-4.

3-3 APPROXIMATE METHOD OF COMPENSATION

In practical applications of this method, it is often impractical and very expensive to use D.F.G.'s to generate the nonlinearity used in the feedforward network. This problem can be overcome by approximating the nonlinearity with two straight line segments. This function is quite simple to generate and the degradation of the response is not drastic. The network used to generate the nonlinear function is shown in Fig. 3-4. If this function is substituted for the nonlinear element in the feedforward network in Fig. 3-3 (page 19), that network can be used to measure the response for various magnitudes of step inputs and to determine the degree to which the response has deteriorated due to the use of this approximate method. The results are shown in Graph 3-5.


GRAPH 3-3
SENSITIVITY OF THE NONLINEAR SYSTEM WITH RESPECT TO TIME
Percentage Overshoot
Percentage Undershoot
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7
2
4
6
8
10
12
14
16
18
20
R_o
T = 1.10 sec.
T = 1.05 sec.
T = 0.95 sec.
T = 0.90 sec.




GRAPH 3-4
SENSITIVITY OF THE
NONLINEAR SYSTEM WITH
RESPECT TO τ.
$\tau = 0.90$
$\tau = 0.95$
$\tau = 1.05$
$\tau = 1.10$
Percentage Overshoot
Percentage Undershoot
5
4
3
2
1
0
-1
-2
-3
-4
-5
2
4
6
8
10
12
14
16
18
20

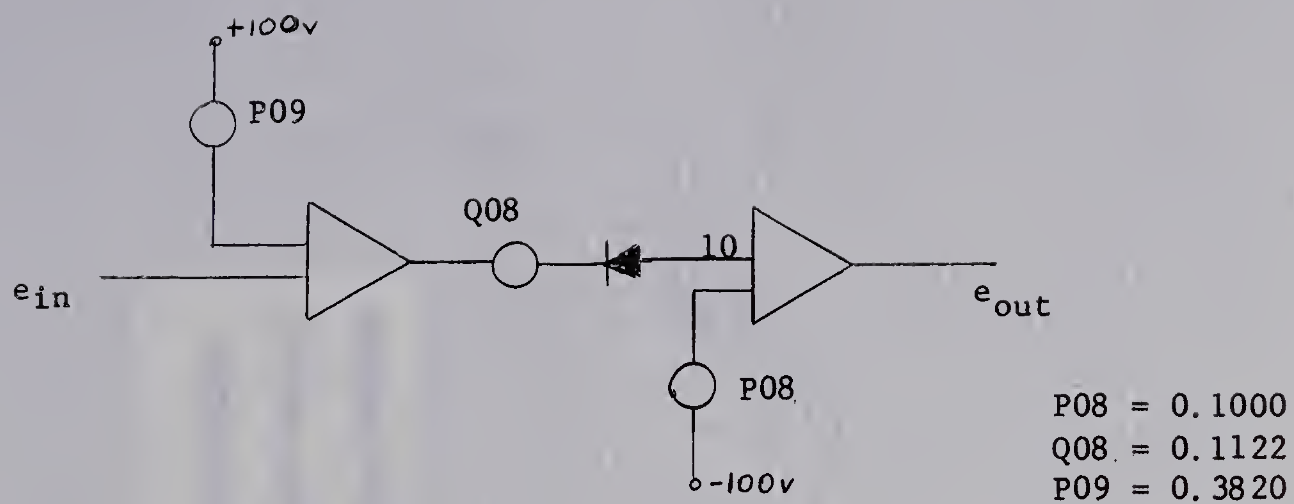


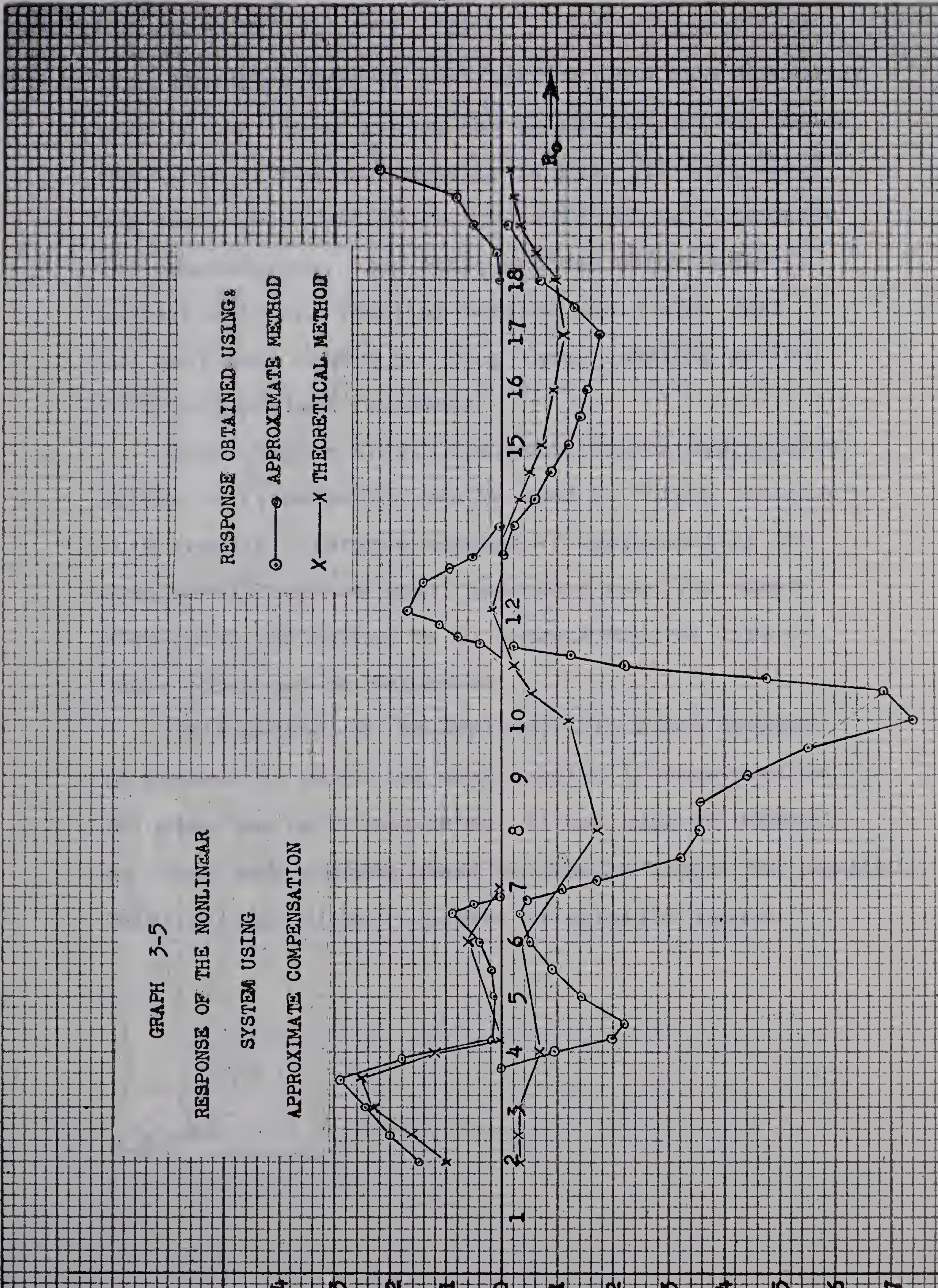
Fig. 3-4

The accuracy of the response has not been reduced drastically. In the range, $7 < R_o < 11$, the function, K_o^f vs. R_o, is in error by the greatest amount. K_o^f is less than the required value and as a result, h_o is too small and the response is an undershoot.

3-4 CONCLUSIONS

The results of this chapter illustrate the necessity for compensation and also how a system with an isolated nonlinearity such as the one used in this example, can be compensated so that almost perfect deadbeat response is obtained, regardless of the magnitude of the step input. The effect of changes in the sampling period, T, and the time constant, τ, is no more serious than for the linear case.

It should be noted that the nonlinearity used in this case is one of the worst possible situations one could expect.


GRAPH 3-5
RESPONSE OF THE NONLINEAR
SYSTEM USING
APPROXIMATE COMPENSATION
RESPONSE OBTAINED USING:
APPROXIMATE METHOD
THEORETICAL METHOD
Percentage Overshoot
Percentage Undershoot
R_o
1 2 3 4 5 6 7 8 9 10 12 15 16 17 18
4 3 2 1 0 1 2 3 4 5 6 7

The saturation type of nonlinearity used here has unity gain in the linear region and in the saturation region, the slope of the nonlinearity is 0.1. If this slope was zero, then this procedure could not be considered for compensation. Any other methods which would require only two sampling periods would also fail. One must then resort to using three sampling periods to obtain minimal response.

This chapter also illustrates how an approximate method of compensation can be used to obtain satisfactory results. Various methods of approximating the nonlinearity can be used, depending upon the range over which the system is to operate and the type of error which can be tolerated.

This method can be applied, with minor changes, to systems in which the nonlinearity is nonsymmetrical. By using one unit-time-delay, it can also be adapted to compensate systems where successive inputs are applied. This includes either positive or negative inputs.

CHAPTER 4

NONLINEAR SYSTEM WITH INTEGRATED NONLINEARITY

In this chapter, as in Chapter 3, a simple, type 1, second order system will be considered. The nonlinearity will be considered to be an integral part of the plant. In most texts or references on the subject, the nonlinearity is considered to be separable from the remainder of the plant. In many practical situations however, this is not so. The block diagram of the system to be considered is shown in Fig. 4-1.

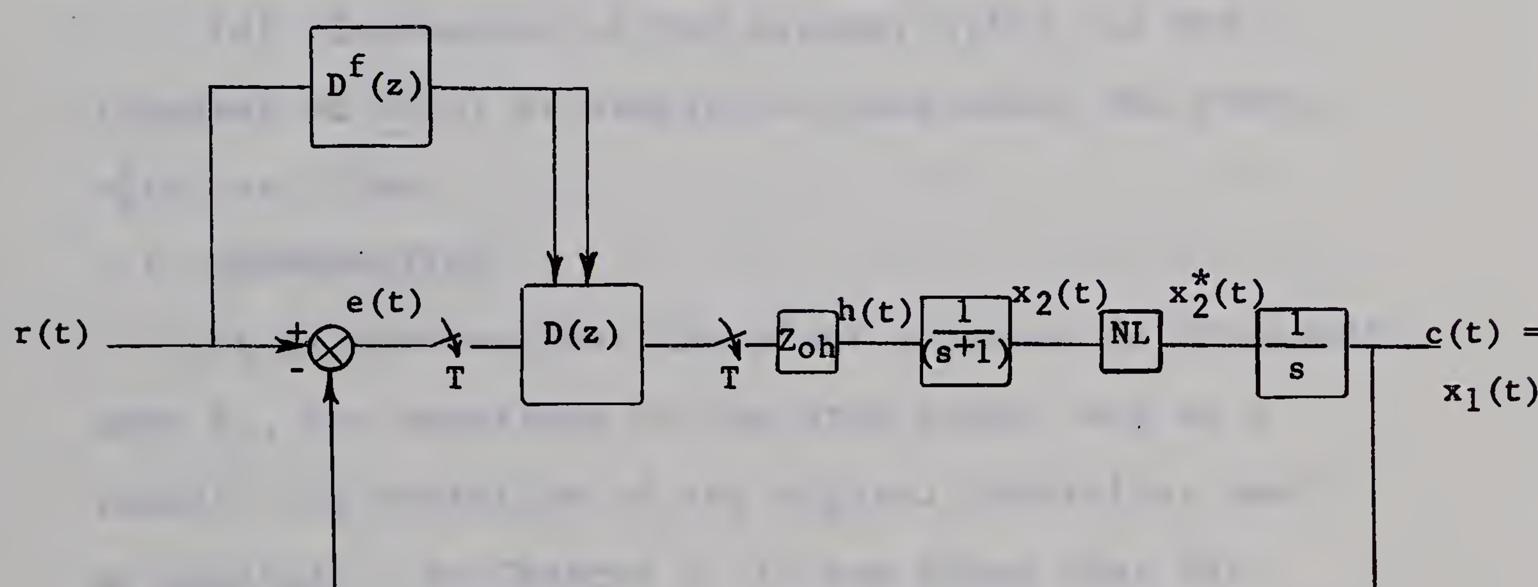


Fig. 4-1

For this system, the overall transfer function can no longer be described by simple linear equations. The input-output relation for each element must be considered separately.

(a) $h(t)$ is a series of steps formed by using the zero-order hold to hold the values, $h(0^+)$, $h(T^+)$, $h(2T^+)$, ... , $h(nT^+)$ through the 'n+1'st sampling period.

(b) $x_2(t)$ is the exponential response to the step function, $h(t)$.

(c) $x_2^*(t)$ is the output of the nonlinearity. This response is a 'distorted exponential' function, the relation between $x_2(t)$ and $x_2^*(t)$ being determined by the nonlinearity.

(d) The output of the system, $x_1(t)$, is the integral of $x_2^*(t)$ or simply the area under the curve, $x_2^*(t)$ vs. time.

4-1 COMPENSATION

It is obvious that the plant response is dependent upon R_o, the magnitude of the step input, and as a result, the operation of the Digital Controller must be analysed. In Chapter 2, it was shown that the transfer function of a digital controller can be described by the following equation.

$$D(z) = \frac{h(0^+) + h(T^+)}{1 + e(T^+)}$$

In this system there are three controlling factors, $h(0^+)$, $h(T^+)$ and $e(T^+)$. For large values of R_o, the two values, $h(0^+)$ and $h(T^+)$, will have to be multiplied

by certain constants to make up for the loss in gain introduced by the nonlinearity. It is also reasonable to assume that the output, $x_1(T)$, will not be equal to 0.5820 R_o and as a result, $e(T^+)$ will not equal $.418R_o$. This parameter will also have to be adjusted to different values for different magnitudes of inputs. Thus it appears as though the three parameters of the Digital Controller will all be nonlinear functions of R_o, the magnitude of the step input. To determine a method to calculate these nonlinear functions, the operation of the system must be considered in a step-by-step fashion. Certain restrictions will then be applied and the unknown values can then be obtained.

Referring to Fig. 4-1, the typical response curves of the various element outputs can be sketched (Fig. 4-3).

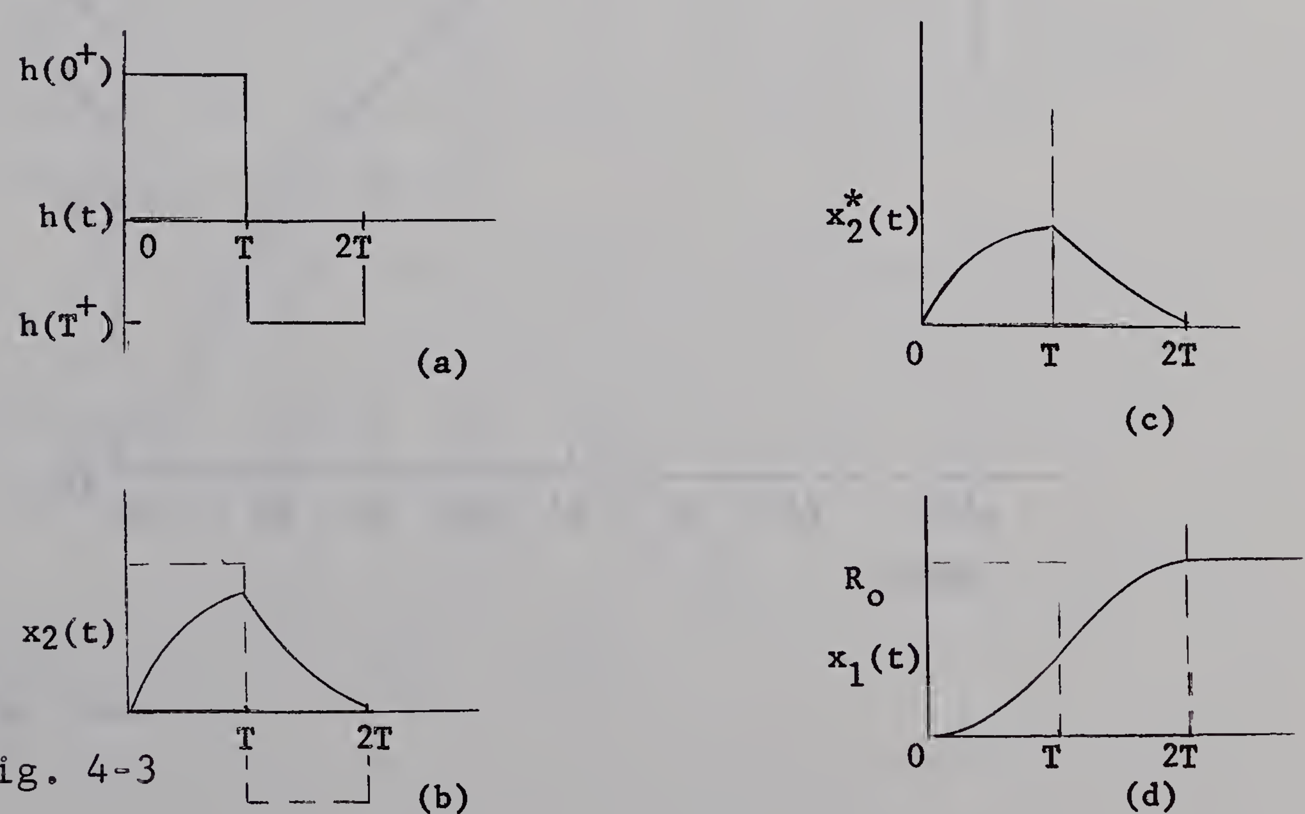


Fig. 4-3

Normally, the relationship between $x_2(t)$ and $x_2^*(t)$ cannot be described by a simple mathematical expression. However, if the nonlinearity is made up of (or approximated by) straight line segments, then separate equations can be written to describe this relationship through each section of the nonlinearity. The sets of points used in making up the nonlinear functions, to be used in the compensating network, can be calculated by repeating the following step-by-step procedure.

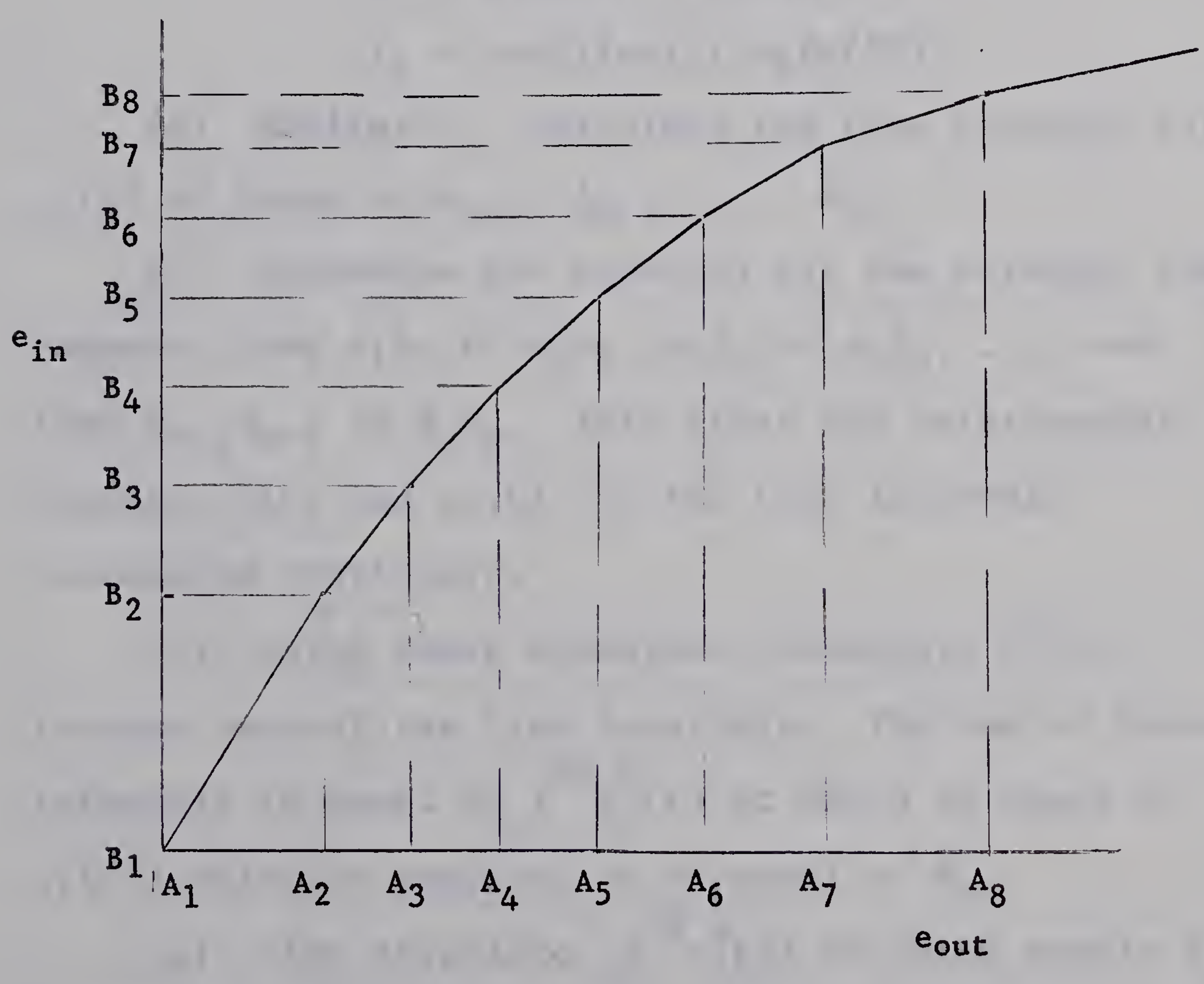


Fig. 4-4

(a) Assume that $h(0^+)$ is of such magnitude that $x_2(T)$ is equal to A_n. (Refer to Fig. 4-4)

Then $A_n = h(0^+)\ (1 - e^{-T})$ and $h(0^+) = A_n/(1 - e^{-T})$

(b) $x_2(2T) = 0$ is one condition which must be satisfied in order to have deadbeat response.

Thus: $$h(T^+) = A_n - h(0^+)$$

(c) Calculate the times required for $x_2(t)$ to rise to each of the values $A_2, A_3, \ldots, A_n$. That is:

$$h(0^+)\ (1 - e^{-t1}) = A_2$$

$$e^{-t1} = (h(0^+) - A_2)/h(0^+)$$

$$t_1 = -\text{antilog}(\ 1 - A_2/h(0^+))$$

(d) Similarly, calculate the time required for $x_2(t)$ to decay to $A_{n-1}, A_{n-2}, \ldots, A_1$.

(e) Determine the equation for the straight line segments from A_1B_1 to A_2B_2, A_2B_2 to A_3B_3, ..., and from $A_{n-1}B_{n-1}$ to A_nB_n. This gives the relationship between $x_2^*(t)$ and $x_2(t)$ for the time intervals calculated previously.

(f) Using these equations, integrate $x_2^*(t)$ through each of the time intervals. The sum of these integrals is equal to $\int_0^{2T} x_2^*(t)\,dt$ which is equal to $x_1(2T)$ which is required to be equal to R_o.

(g) Also calculate $\int_0^{T} x_2^*(t)\,dt$ which equals $x_1(T)$. From this, $e(T) = R_o - x_1(T)$.

(h) $R_oK_oK_o^f = h(0^+)$ $\quad\quad$ $K_o^f = h(0^+)/\ R_oK_o$

$R_oe_1K_1K_1^f = h(T^+)$ $\quad\quad$ $K_1^f = h(T^+)/\ R_oK_1e_1$

$R_oe_1 \ = \ e(T^+)$ $\quad\quad$ $e_1(R_o) = e(T^+)/\ R_o$

These values form one set of points for the three curves relating K_o^f, K_1^f and e_1 to R_o, and are valid for only one value of R_o. This whole series of calculations can be done very simply by using the digital computer. The computer program, along with the explanation of the notations used, is shown in Appendix B.

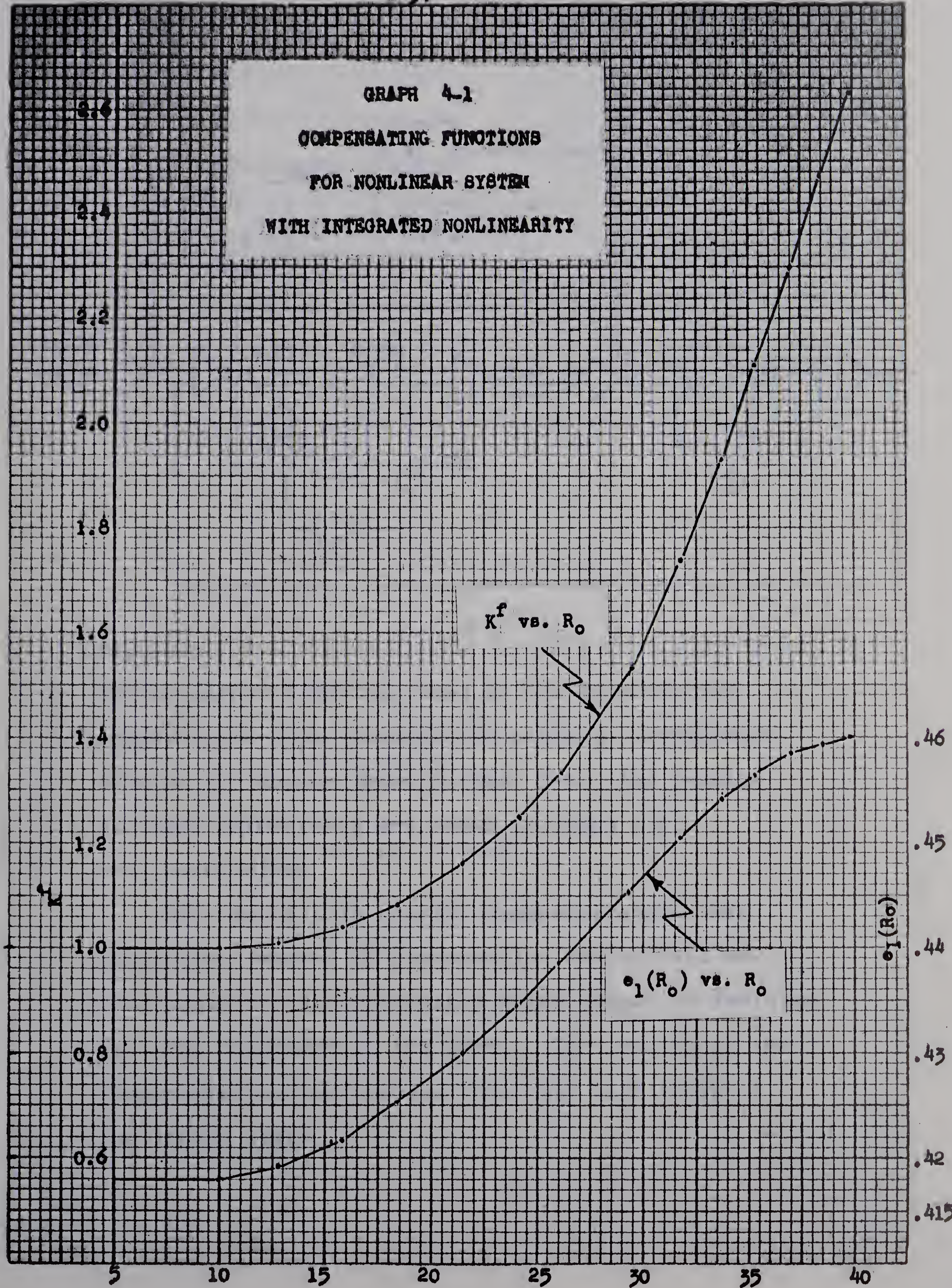
The results of this program give the values of K_o^f, K_1^f and e_1 required for various values of R_o. The values of K_o^f and K_1^f were found to be equal for each value of R_o. This means that the outputs of the original digital controller, R_oK_o and $e_1K_1R_o$, need to be multiplied by the same correction factor, K^f, for each value of R_o. The functions, K^f and e_1 vs. R_o, are plotted on Graph 4-1.

4-2 SIMULATION

Using the above results, a system can be simulated to test the response for various magnitudes of step inputs. The system is shown in Fig. 4-5.

4-3 RESULTS

Using the system shown in Fig. 4-5, the response to various magnitudes of step inputs was measured. To illustrate the effectiveness of this feedforward method of compensation, the same system with no compensation was tested.


GRAPH 4-1
COMPENSATING FUNCTIONS
FOR NONLINEAR SYSTEM
WITH INTEGRATED NONLINEARITY
K^f vs. R_o
$e_1(R_o)$ vs. R_o
K^f
$e_1(R_o)$
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
.46
.45
.44
.43
.42
.415
5
10
15
20
25
30
35
40

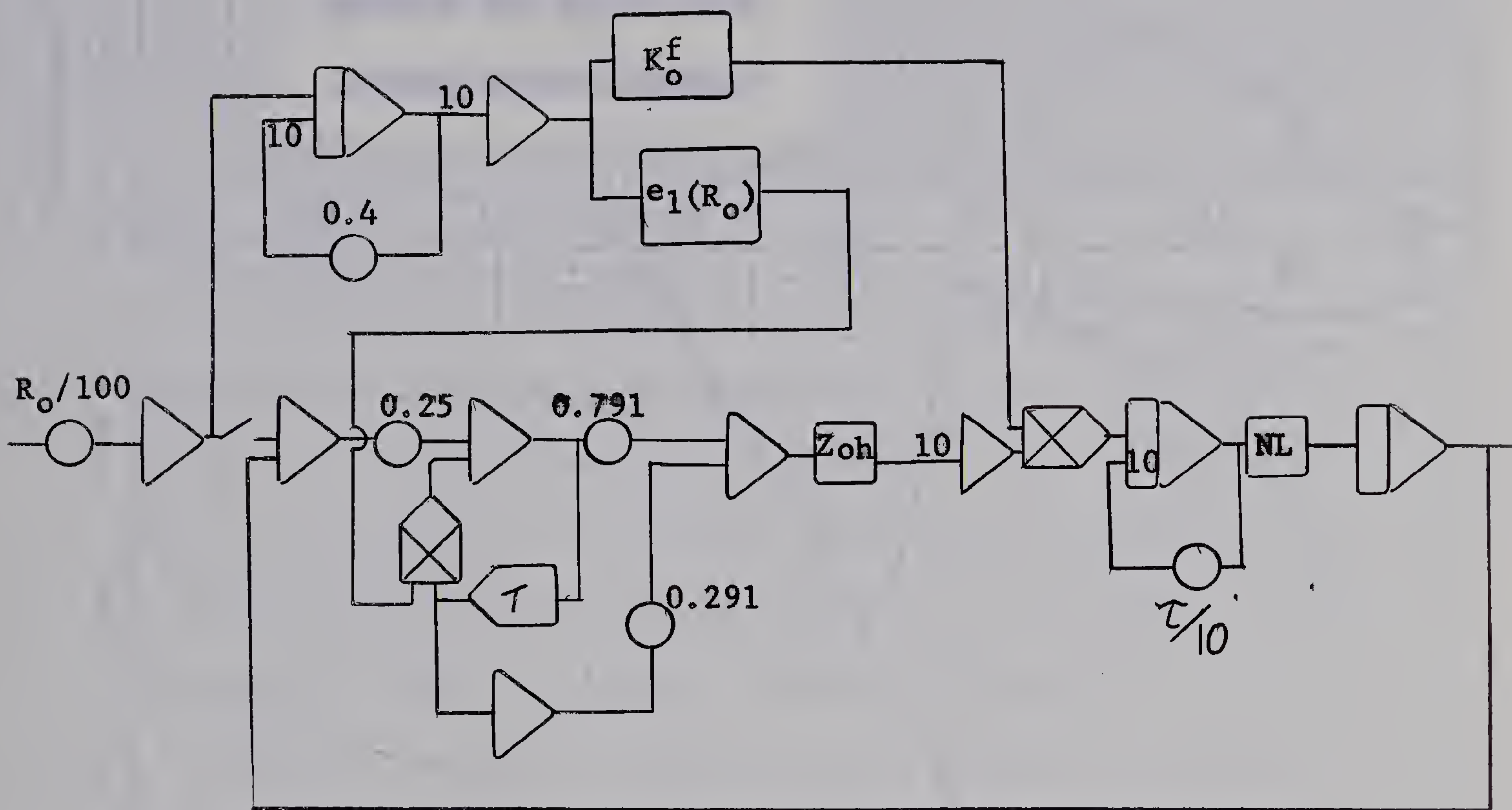


Fig. 4-5

The nonlinear functions used in the compensating network were then approximated using an arrangement similar to the one shown in Fig. 3-4. The results of these three tests are shown in Graph 4-2.

4-4 CONCLUSIONS

Results of this chapter demonstrate how this feedforward method of compensation can be used to compensate for a nonlinearity which cannot be isolated from the remainder of the plant. The results of the compensated system compare favorably with those of the linear system.

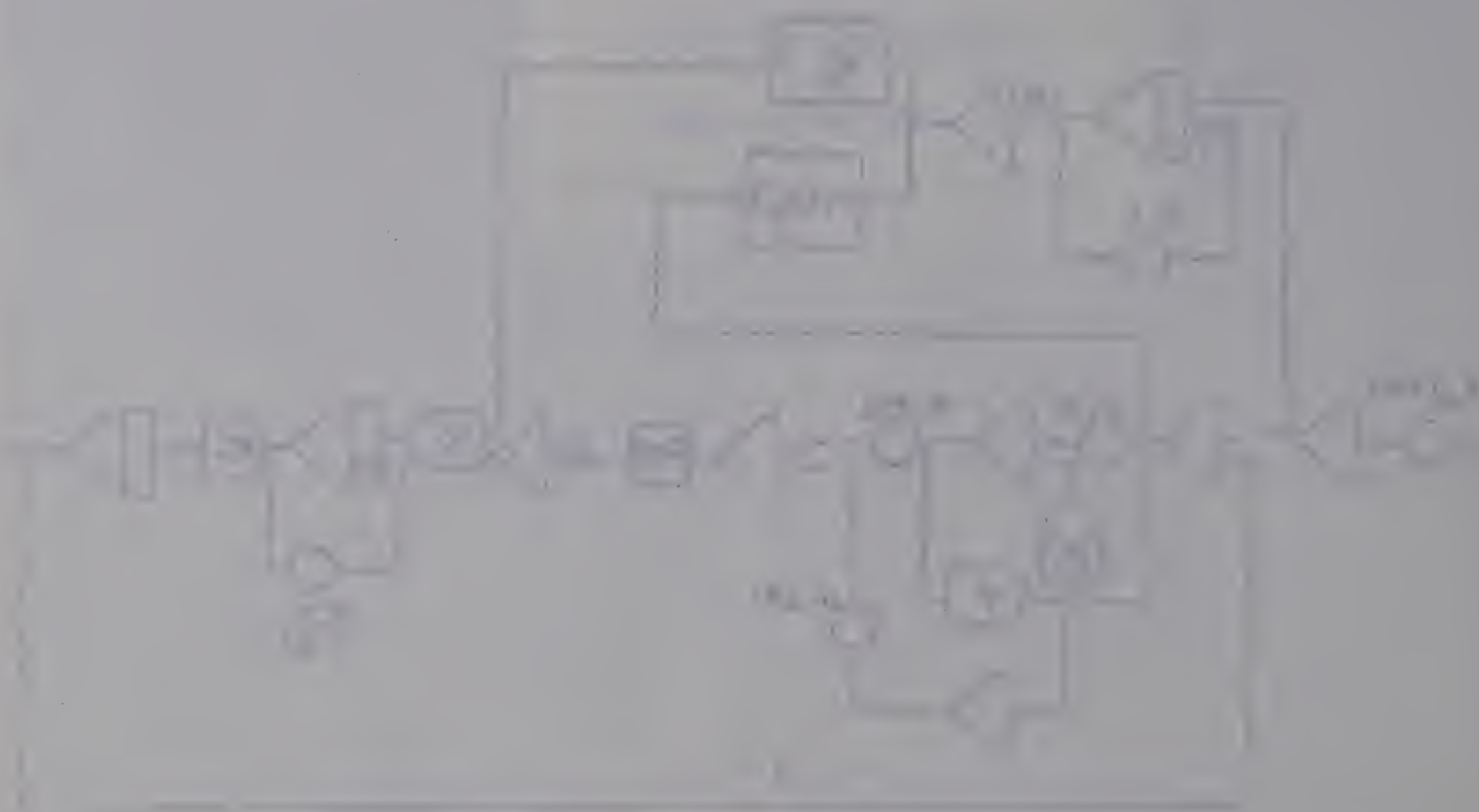
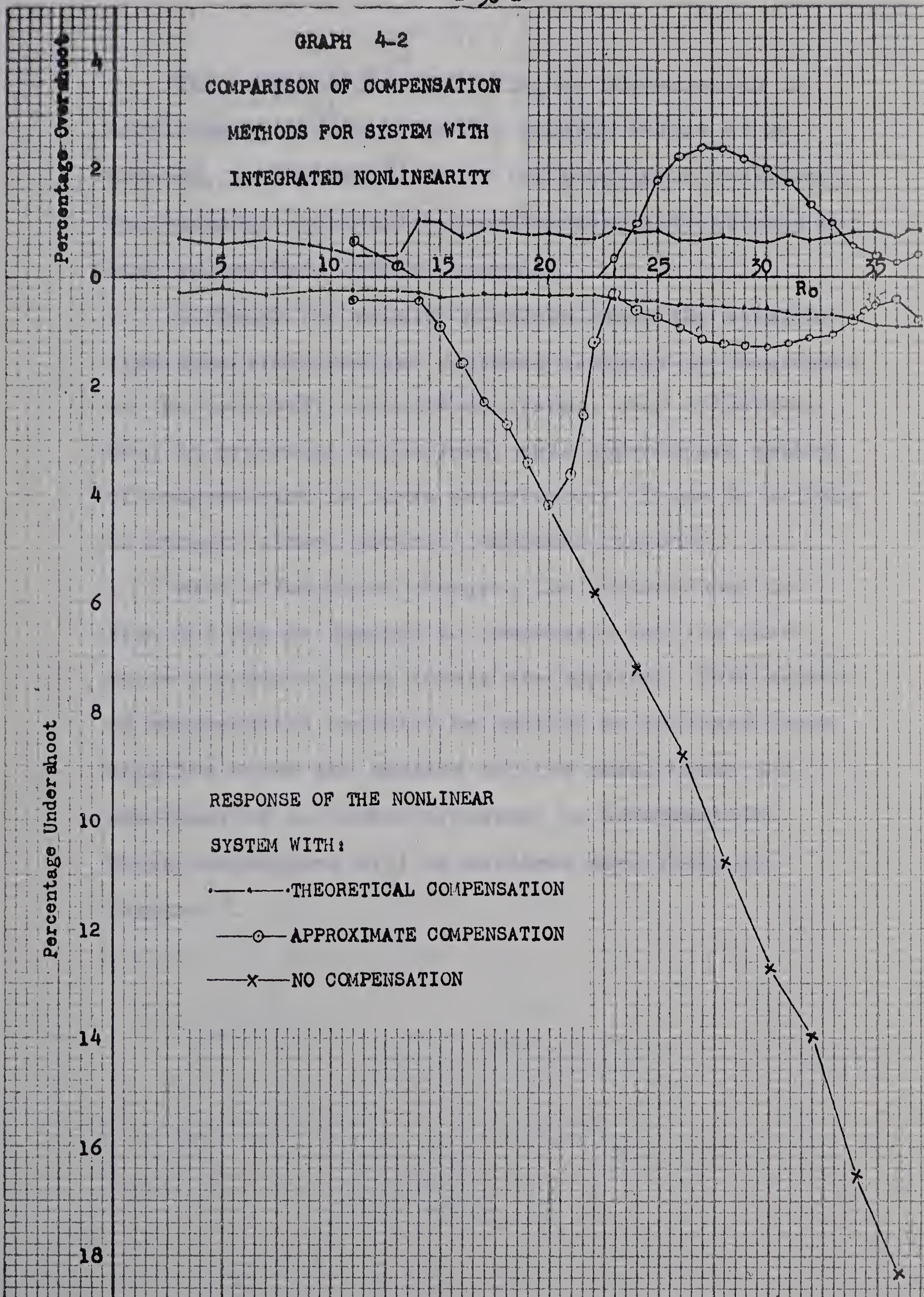


Fig. 4-1

The nonlinear functions used in the compensating network were those characteristics) similar to those similar to the one shown in Fig. 3-4. The results of three input types are shown in Table 4-1.

4.7 CONCLUSIONS

Results of this chapter demonstrated that this compensating network can be used to compensate for a nonlinearity which cannot be isolated from the remainder of the system. The results of the compensation are particularly with those of the linear system.


GRAPH 4-2
COMPARISON OF COMPENSATION
METHODS FOR SYSTEM WITH
INTEGRATED NONLINEARITY
Percentage Overshoot
Percentage Undershoot
4
2
0
2
4
6
8
10
12
14
16
18
5
10
15
20
25
30
35
Ro
RESPONSE OF THE NONLINEAR
SYSTEM WITH:
THEORETICAL COMPENSATION
APPROXIMATE COMPENSATION
NO COMPENSATION

The response obtained using no compensation is sufficiently accurate for the smaller values of R_o. However, as the magnitude of the step input increases, the response continually deteriorates until it does not even approximate minimal response.

Although the results obtained using the straight line approximations are degraded slightly in comparison to the correctly compensated system, they illustrate how, in practical situations, this approximate method of compensation is quite satisfactory if one is willing to accept 'almost perfect' deadbeat response.

With a few minor changes, the system shown in Fig. 4-5 can be adapted to compensate for the case where successive step inputs are applied. This method of compensation can also be applied to the case where negative steps are applied and the cases where the nonlinearity is either symmetric or nonsymmetric. These adaptations will be outlined more fully in Chapter 7.

CHAPTER 5

DESIGN OF COMPENSATING NETWORKS USING AN EXPERIMENTAL METHOD

The results of Chapter 4 illustrate how effective the feedforward method is in compensating for certain types of nonlinearities in sampled-data systems. The only problem arises in deriving the nonlinear functions which are to be used in the feedforward network. In most cases, a digital computer may not be available to do the extensive calculations required. If the input-output characteristic of the plant nonlinearity is not known or if the time constant in the $1/(s+\tau)$ term cannot be determined exactly, then the method outlined in Chapter 4 cannot be used.

An alternative method of deriving the 'feedforward functions' will be outlined in this chapter. The experimental method used here is simplified by utilizing the 'Repetitive Operation' (Rep. Op.) Mode of the analog computer. High speed electronic comparators are used to apply a series of steps to the open loop system. The basic circuit used is shown in Fig. 5-1.

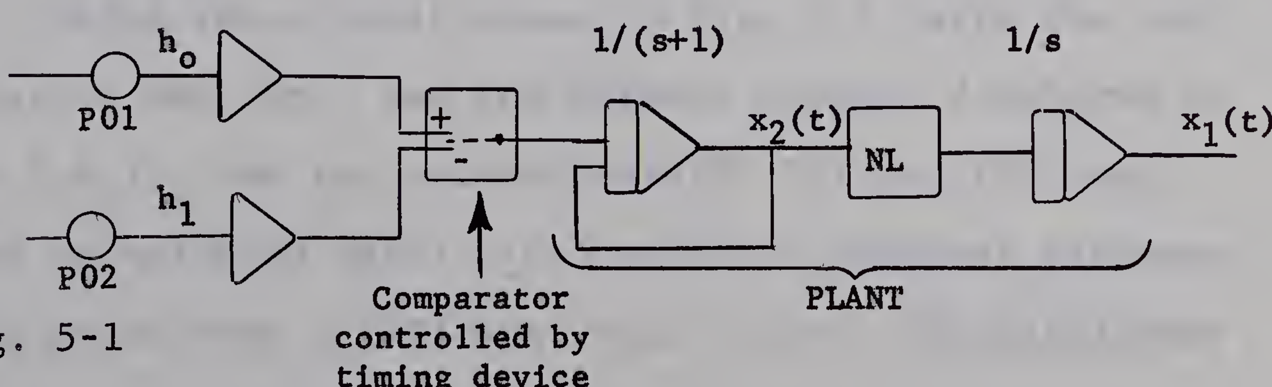


Fig. 5-1

CHAPTER 5

DESIGN OF COMPENSATING NETWORK

BASED ON EXPERIMENTAL RESULTS

The results of Chapter 4 illustrate the efficacy [illegible] of nonlinearities in feedback control systems. The only problem [illegible] which are to be used in the compensated networks. [illegible] Chapter 4 cannot be used.

An alternative method of deriving the 'Equivalent functions' will be described in this chapter. The experimental method used here is [illegible] the 'Describing Function' [illegible] analog computer. [illegible] was used to apply a series of steps to the open loop system. The basic circuit used is shown in Fig. 5-1.

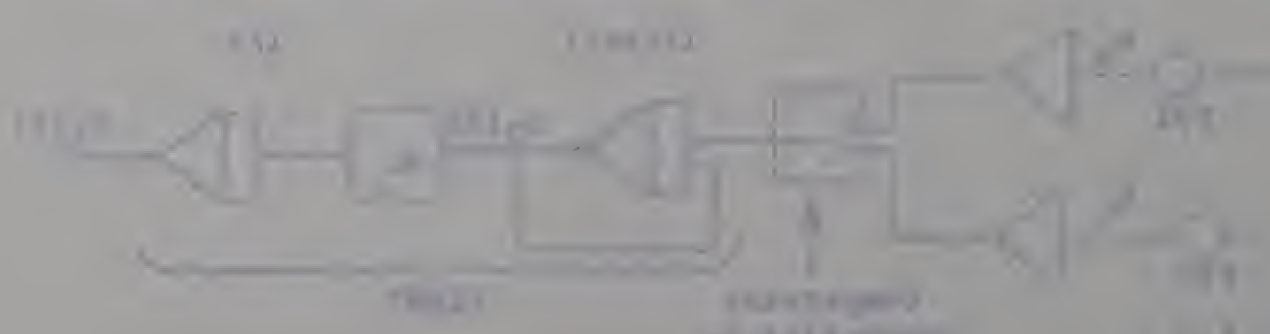


FIG. 5-1

5-1 REPETITIVE OPERATION MODE

The Rep. Op. mode of the analog computer is a special mode, in which the response of any circuit is speeded up by a factor of 500. The computer automatically switches from 'Initial Conditions' Mode to 'Operate' Mode. The operate time can be selected as 10, 20, 40, or 80ms. duration.

5-2 TIME SCALING

In this example, the system which is simulated on the computer can be slowed down by a factor of 20, so that in Rep. Op., one second of real time corresponds to 20/500 or 40ms. of computer time. The entire response of the system, corresponding to two sampling periods, can then be displayed in 80 ms. Including reset time, the computer takes approximately 100 ms. of real time for each response. The response is then repeated at a rate of about 10 times per second. The various outputs can then be displayed on a C.R.T. and appear as continuous curves.

5-3 RESULTS

Using the circuit shown in Fig. 5-1, with the computer in Rep. Op., and the various outputs displayed on the C.R.T., the two potentiometers, P01 and P02, can then be adjusted until $x_1(t)$ exhibits deadbeat response. This means that $x_2(2T)$ must equal zero. The magnitudes

of the pulses, $h(0^+)$ and $h(T^+)$, as well as $x_1(T)$ can then be measured. This process is then repeated for several values of $x_2(2T)$. With the values of h_o, h_1, $x_1(T)$ and $x_1(2T)$ recorded for each case, K_o^f, K_1^f and e(T) can then be calculated.

$$h(0^+) = K_oR_oK_o^f \qquad K_o^f = h(0^+)/K_oR_o$$

$$h(T^+) = e_1K_1R_oK_1^f \qquad K_1^f = h(T^+)/e_1K_1R_o$$

$$e(T^+) = e_1R_o \qquad e_1(R_o) = e(T^+)/R_o$$

The results of Chapter 4 indicate that K_o^f should be equal to K_1^f. The average value can be used in this case to obtain a value of K^f. The actual computer diagram used to illustrate this experimental method is shown in Fig 5-2, the results being displayed on Graph 5-1.

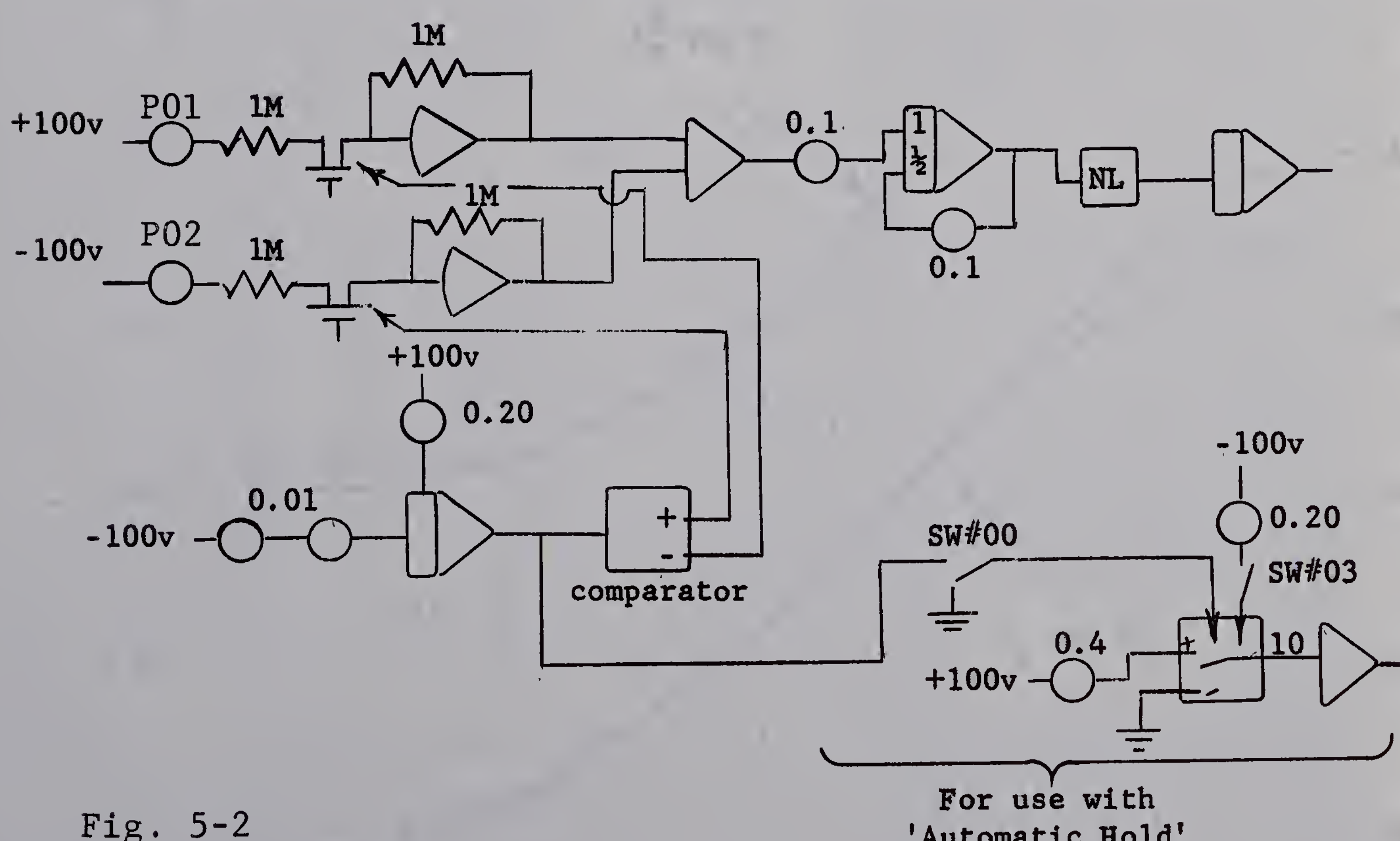
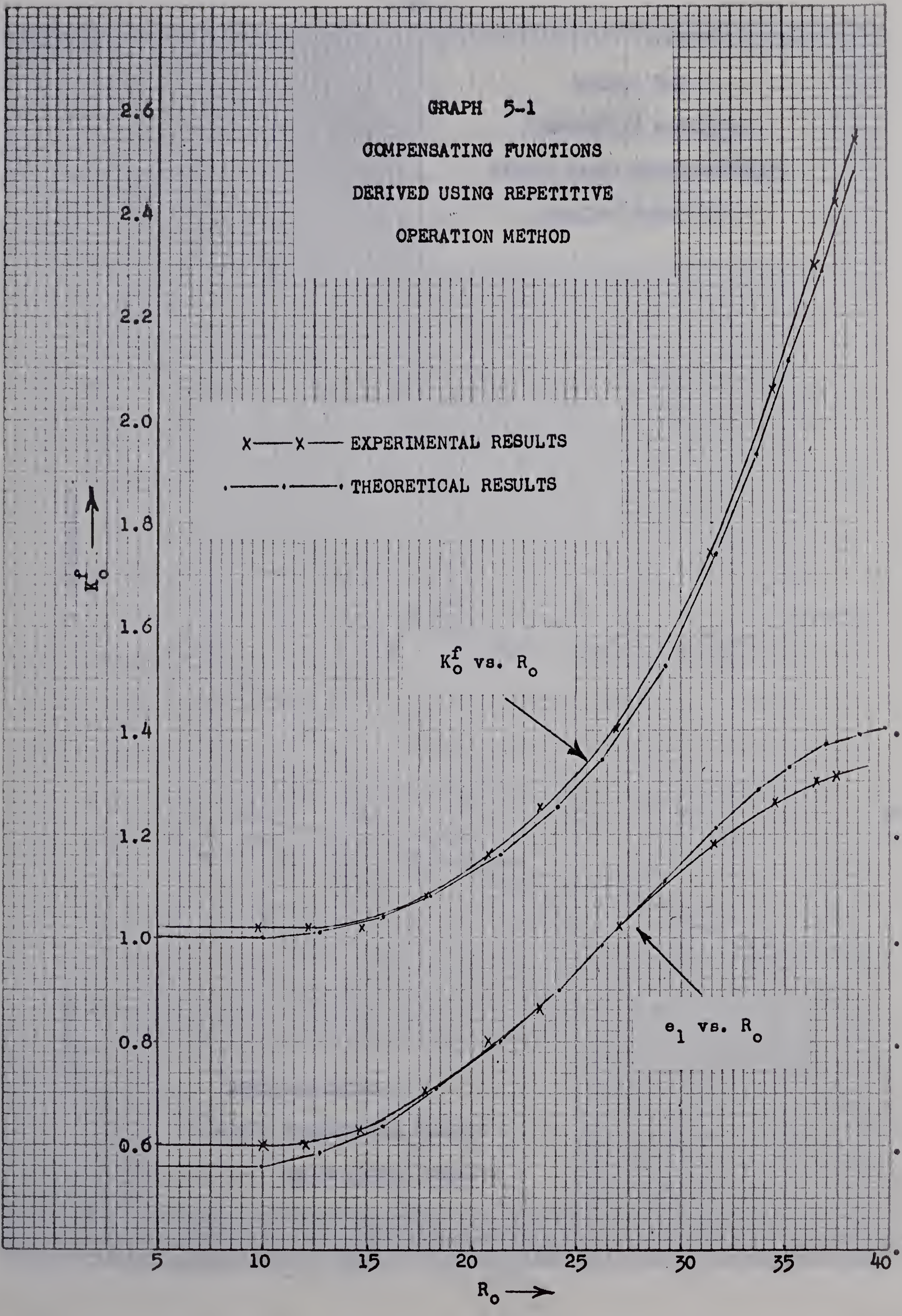
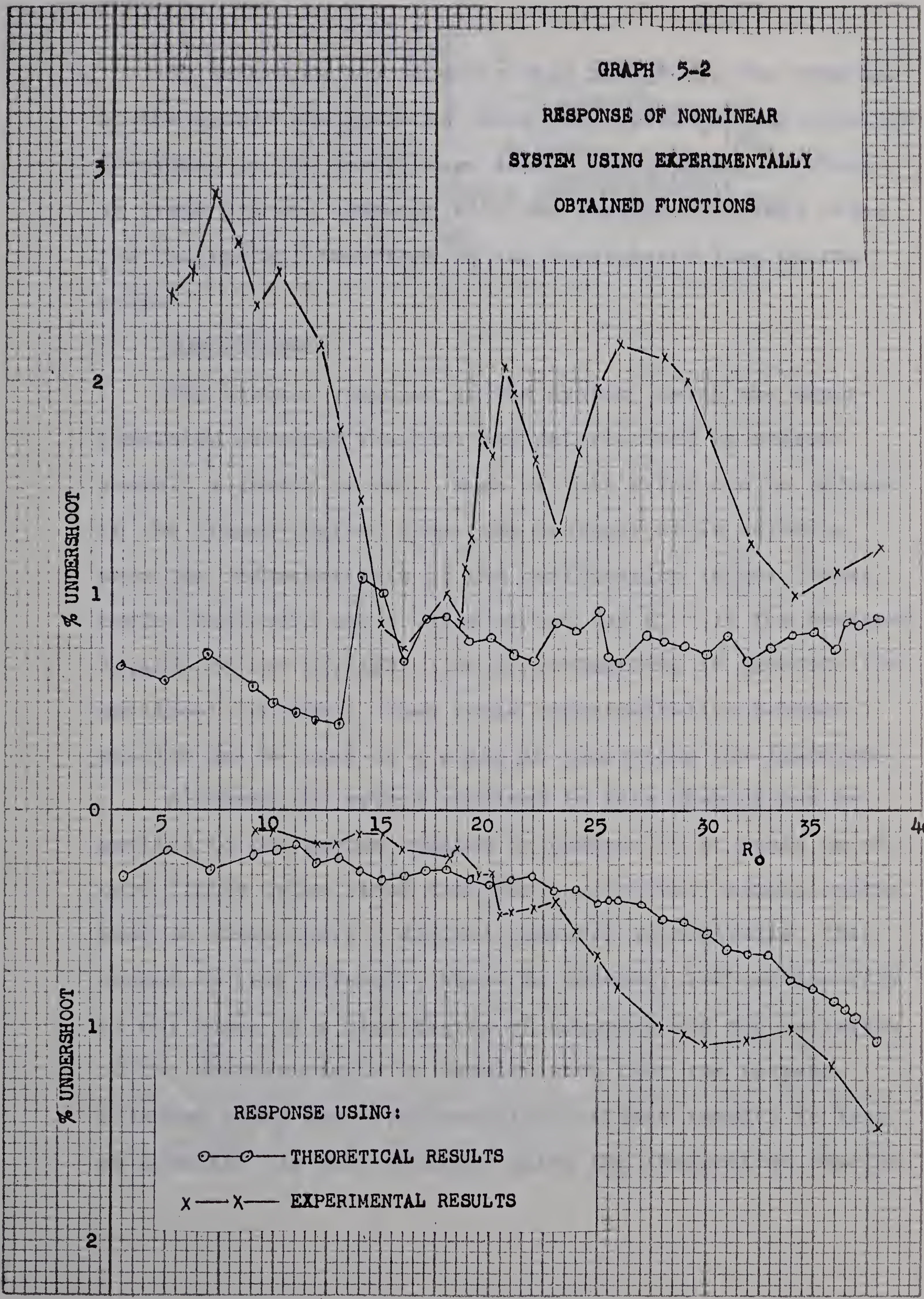


Fig. 5-2


GRAPH 5-1
COMPENSATING FUNCTIONS
DERIVED USING REPETITIVE
OPERATION METHOD
EXPERIMENTAL RESULTS
THEORETICAL RESULTS
K_o^f vs. R_o
e_1 vs. R_o
K_o^f
e_1
R_o
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
.46
.45
.44
.43
.42
.41
5
10
15
20
25
30
35
40


GRAPH 5-2
RESPONSE OF NONLINEAR
SYSTEM USING EXPERIMENTALLY
OBTAINED FUNCTIONS
% UNDERSHOOT
3
2
1
0
5
10
15
20
25
30
35
40
R
% UNDERSHOOT
1
2
RESPONSE USING:
THEORETICAL RESULTS
EXPERIMENTAL RESULTS

To determine how valuable this method is, the response of the system was measured using the experimentally-obtained functions in the feedforward loop. The response is shown in Graph 5-2 and compared with the response obtained using the theoretical functions in the feedforward loop of the system.

5-4 CONCLUSIONS

The overall response of the system, using the experimentally obtained results, has deteriorated by approximately a factor of two. Much of this error can be reduced in the linear region, since the designer would normally know the characteristic of the nonlinearity in the linear region and would use a value of one for K_o^f. If the designer intends to use straight line approximations to generate the nonlinear functions, then these experimentally-obtained results can be used as a guide in generating the functions.

Although the method outlined in this chapter can be applied to the system studied in Chapter 3, it would be of very little value since that was a relatively simple system. Even in cases where a digital computer is available, this method is very effective since in general, the nonlinearity is not known to a high degree of accuracy and the variation of the characteristic is usually such that the response obtained using the experimentally-obtained results is just as accurate as that obtained using the theoretical results.

To demonstrate the validity of this method, the response of the system was computed using the experimentally-obtained functions of the feedforward loop. The response is shown in Figure 5-3 and compared with the response obtained with the theoretical functions in the feedforward loop of the system.

5.4 CONCLUSIONS

The overall response of the system, using the experimentally obtained results, has been improved by approximately a factor of ten. Most of this error can be reduced in the linear region, since the original would normally have the characteristic of the nonlinearity in the linear region and would be a value of one (in %). If the designer chooses to use straight line approximations to generate the nonlinear functions, then these approximations would usually serve as a guide to generating the functions.

Although the method outlined in this chapter was not applied to the system discussed in Chapter 3, it would be of very little value since that was a relatively simple system. Since the cases where a digital computer is available, this method is very simple to apply. In general, the nonlinearity in the system is not known to a high degree of accuracy, and the variation of the characteristic is usually such that the response obtained using the experimentally-obtained results is just as accurate as that obtained using the theoretical results.

CHAPTER 6

OPTIMIZATION OF THE SYSTEM USING THREE SAMPLING PERIODS

In the systems which have been studied in the previous chapters, the range of inputs, R_o, was appropriately chosen so that the input to the non-linear element covered a convenient range of the non-linear region. In practical systems, the nonlinearity may be such that the saturation becomes very severe and the output may even approach a finite maximum (see Fig. 6-1). This situation would require excessively large plant inputs in order to obtain the required output. In cases of complete saturation, the required output may never be obtained.

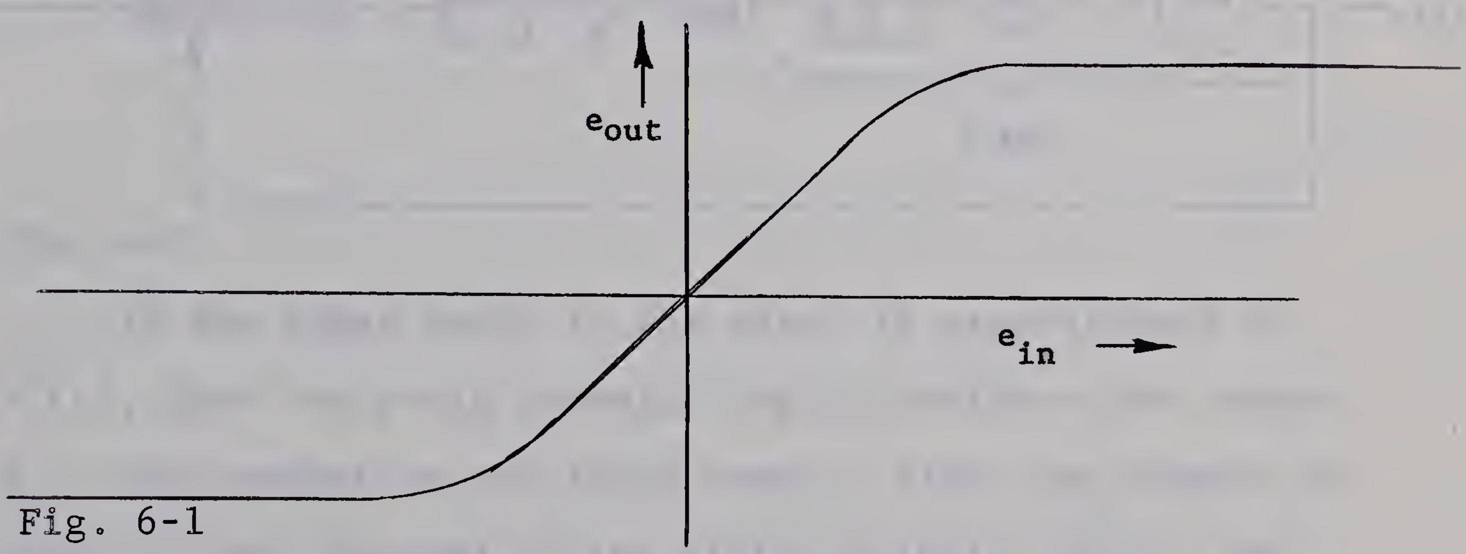


Fig. 6-1

One solution to the problem is to increase the sampling period. This however degrades the response for smaller magnitudes of inputs. The other obvious solution is to use three sampling periods for the larger inputs and the regular two periods for the smaller inputs.

Since using three sampling periods necessarily means the addition of one variable, a number of alternatives are available in the design of the digital controller. In general, one would normally try to optimize one or more items in the system response, the item to be optimized depending upon the system under consideration.

6-1 SOLUTION

In this chapter, the system used in Chapter 4 will again be considered (see Fig. 6-2).

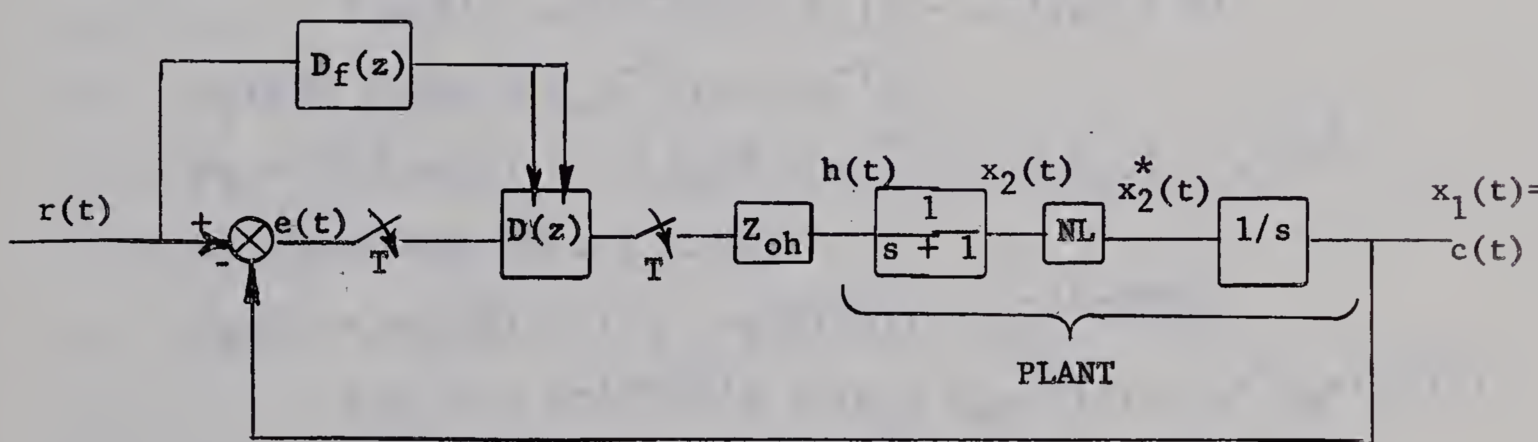
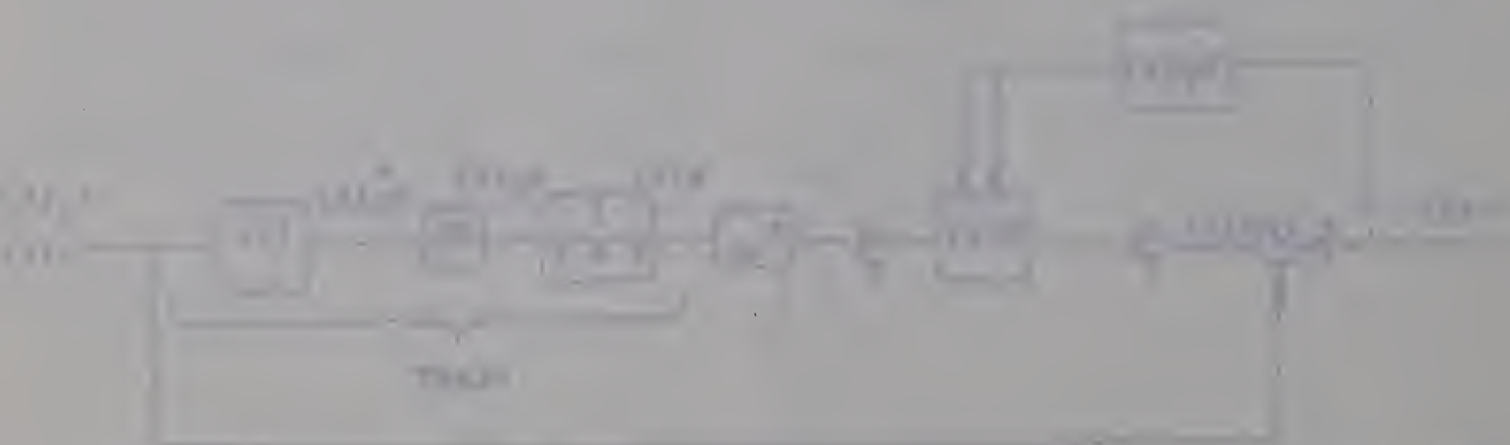


Fig. 6-2

If the power input to the plant is proportional to $h^2(t)$, then one would normally try to maximize the output, R_o, with respect to the input power. Since the output is equal to the integral of the state variable, $\overset{*}{x}_2(t)$, and since $\overset{*}{x}_2(t)$ is related to $x_2(t)$ through the nonlinear element, consider first the integral of $x_2(t)$ for various values of h(t) (see Fig. 6-3). In the range $0 \leq t < T$:

$$x_2(t) = h_o(1 - e^{-t})$$

$$A_1 = h_o\int_0^T (1 - e^{-t})dt = h_o(t + e^{-t})\Big]_0^T = h_o(T + e^{-T} - 1)$$



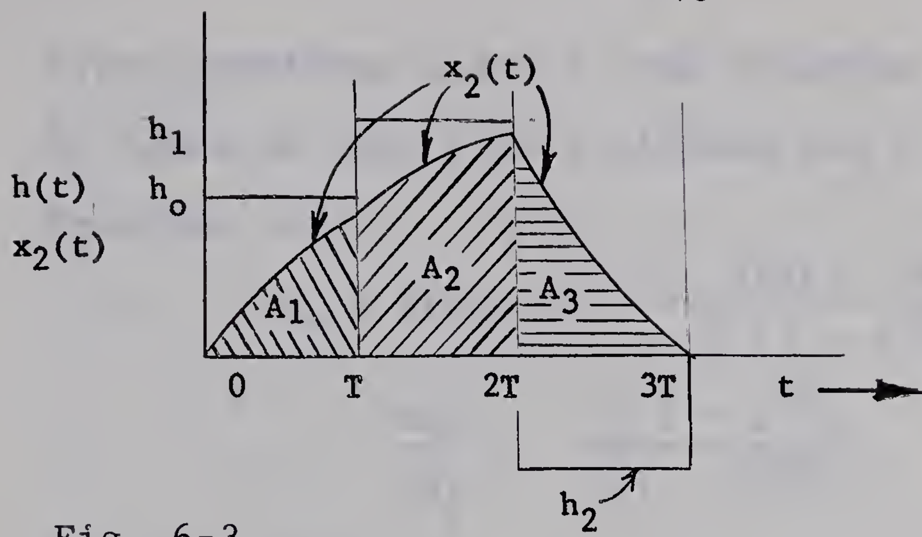


Fig. 6-3

In the range $T < t < 2T$:

$$x_2(t) = x_2(T) + (h_1 - x_2(T))(1 - e^{-(t-T)})$$
$$= h_1(1 - e^{-(t-T)}) + h_o(1 - e^{-T})e^{-(t-T)}$$
$$x_2(2T) = (h_1 + h_o e^{-T})(1 - e^{-T})$$
$$A_2 = \int_T^{2T} x_2(t)\,dt = h_1(T + e^{-T} - 1) + h_o(1 - e^{-T})^2$$

In the range $2T < t < 3T$:

$$x_2(t) = x_2(2T) + (h_2 - x_2(2T))(1 - e^{-(t-2T)})$$
$$= h_2(1 - e^{-(t-2T)}) + (h_1 + h_o e^{-T})(1 - e^{-T})e^{-(t-2T)}$$

However, the requirement for deadbeat response is that $x_2(3T) = 0$. Thus:

$$h_2(1 - e^{-T}) + (h_1 + h_o e^{-T})e^{-T}(1 - e^{-T}) = 0$$
$$h_2 = -(h_1 + h_o e^{-T})\, e^{-T}$$
$$x_2(t) = (h_1 + h_o e^{-T})\,(e^{-(t-2T)} - e^{-T})$$
$$A_3 = \int_{2T}^{3T} x_2(t)\,dt = (h_1 + h_o e^{-T})\,(1 - Te^{-T} - e^{-T})$$

The total power is $A_T = A_1 + A_2 + A_3$

$$= h_o T(1 - e^{-2T}) + h_1 T(1 - e^{-T}) \qquad \text{Eqn. 6-1}$$

The total power into the plant is assumed to be proportional to $(h_o^2 + h_1^2 + h_2^2) = h_o^2 + h_1^2 + (h_1 + h_o e^{-T})^2 e^{-2T} \propto P$ Eqn. 6-2

Using equations 1 and 2, the relation between h_1 and h_o can be found so that P is a minimum for a constant area, A_T.

From Eqn. 6-1:

$$h_o = f(h_1) = \frac{C - h_1 T(1 - e^{-T})}{T\,(1 - e^{-2T})}$$

$$\frac{dh_o}{dh_1} = - \frac{(1 - e^{-T})}{(1 - e^{-2T})}$$

From Eqn. 6-2:

$$\frac{dP}{dh_1} = \frac{\partial P}{\partial h_1}\frac{dh_1}{dh_1} + \frac{\partial P}{\partial h_o}\frac{dh_o}{dh_1} = 0$$

Solving, $h_1 = h_o(1 - e^{-T})$

This is the condition to be satisfied in order that, for a specified value of A_T, the input power is minimized. Alternatively, for a specified power input, the maximum output is obtained if $h_1 = h_o(1 - e^{-T})$. This means that $x_2(t)$ is constant through the second sampling period. That is,

$$x_2(t) = h_o(1 - e^{-T}) \text{ for } T < t < 2T.$$

Typical response curves are shown in Fig. 6-4.

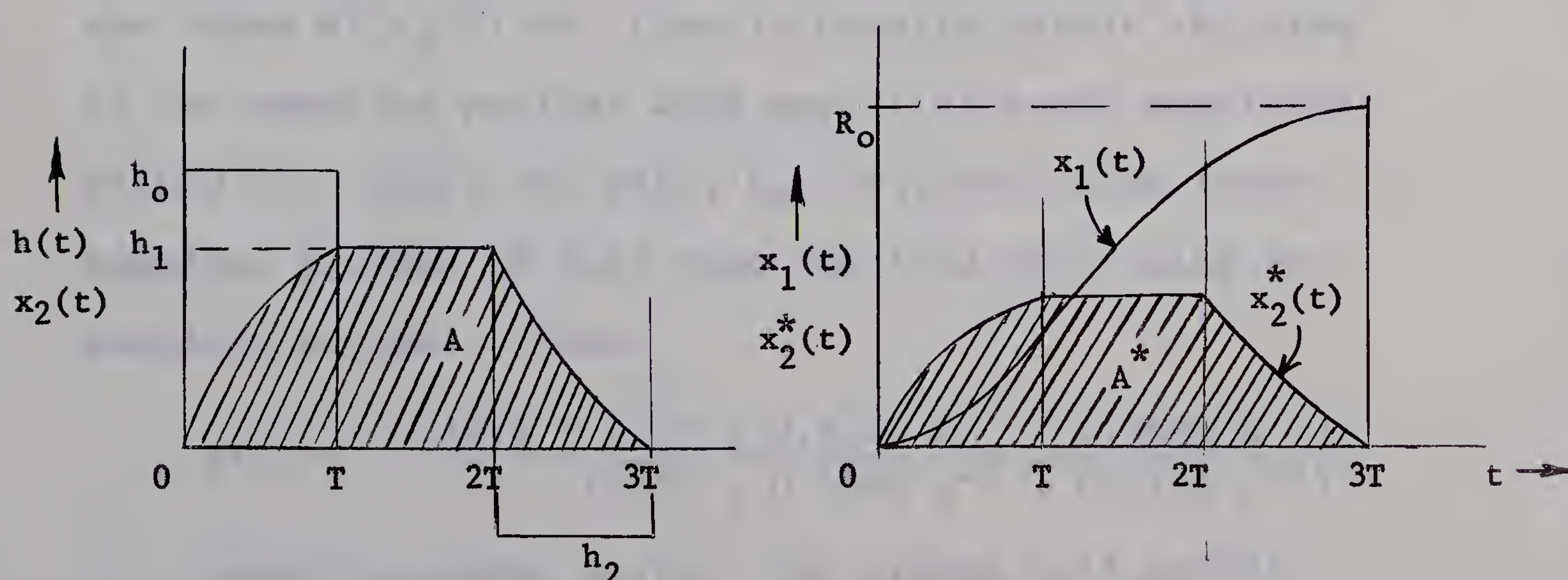


Fig. 6-4 (a) (b)

Using Theorems 1 and 2, the relations between u_1 and h_T can be found at time T to a maximum for a constant value, h_T.

From Eq. 4.14:

$$\frac{(1 - h_T)(1 - e^{-T})}{(1 - e^{-T})} \qquad h_1(T)$$

$$\frac{\partial h_2}{\partial u_1} = \frac{(1 - e^{-T})}{(1 - e^{-T})}$$

From Eq. 4-2:

$$\frac{\partial F}{\partial u_1} = \frac{\partial F}{\partial h_1}\frac{\partial h_1}{\partial u_1} + \frac{\partial F}{\partial h_2}\frac{\partial h_2}{\partial u_1} = 0$$

giving, $\quad u_1 = h_T(1 - e^{-T})$

This is the condition for an optimal ... i.e. for a specified value of h_T the final power is maximized. Alternatively, for a specified power input, the maximum output is obtained if $u_1 = h_T(1 - e^{-T})$. This means that $u_1(t)$ is constant through the second treating period. And so,

$u_1(t) = h_T(1 - e^{-T})$ for $T < t < 2T$.

Typical response curves are shown in Fig. 4-5.

Fig. 4-5 (a) (b)

To study the relationship between $x_2(t)$ and $x_2^*(t)$, let $\int_0^{3T} x_2(t)\,dt = A$ and $\int_0^{3T} x_2^*(t)\,dt = A^*$. For a given value of A, the maximum value of A^* occurs when the response is as shown in Fig. 6-4 (i.e. $h_1 = h_o(1 - e^{-T})$). This means that $x_1(3T) = R_o$ is maximized. In other words, for a specific value of R_o, the input power to the plant and the maximum value of $x_2(t)$ (and also $x_2^*(t)$) can both be minimized by applying the condition, $h_1 = h_o(1 - e^{-T})$.

6-2 SIMULATION

Using the conditions,

$$h_1 = h_o(1 - e^{-T}) \quad \text{and}$$

$$h_2 = -(h_1 + h_o e^{-T})\, e^{-T} = -h_o e^{-T},$$

D(z) can readily be written as,

$$D(z) = \frac{h_o + h_o(1 - e^{-T})z^{-1} + (-h_o e^{-T})\, z^{-2}}{1 + e(T^+)\, z^{-1} + e(2T^+)\, z^{-2}}$$

By referring to Fig. 6-4a, it is noticed that for the system using three sampling periods, the area under the curve of $x_2(t)$ vs. time is exactly double the area if two sampling periods were used (for equal magnitudes of $h(0^+)$). Thus, the gain, K_o, required using three sampling periods is ½ of that required when using two sampling periods. Thus:

$$D(z) = \frac{.7910\ (1.00 + 0.6320\ z^{-1} - 0.3680\ z^{-2})}{(1.00 + 0.7090\ z^{-1} + 0.2140\ z^{-2})}$$

Using the above design, the system will exhibit deadbeat response in three sampling periods, as long as the operation of the system is in the linear region.

It can also be seen from Fig. 6-4(a) that the operation of the system is linear for all magnitudes of step inputs from 0 to 20 volts. For inputs larger than this, feed-forward compensation as discussed in Chapter 4 is necessary.

If the operation of the system is limited to inputs in the range, 0 to 20 volts, then the following system, shown in Fig. 6-5, can be used.

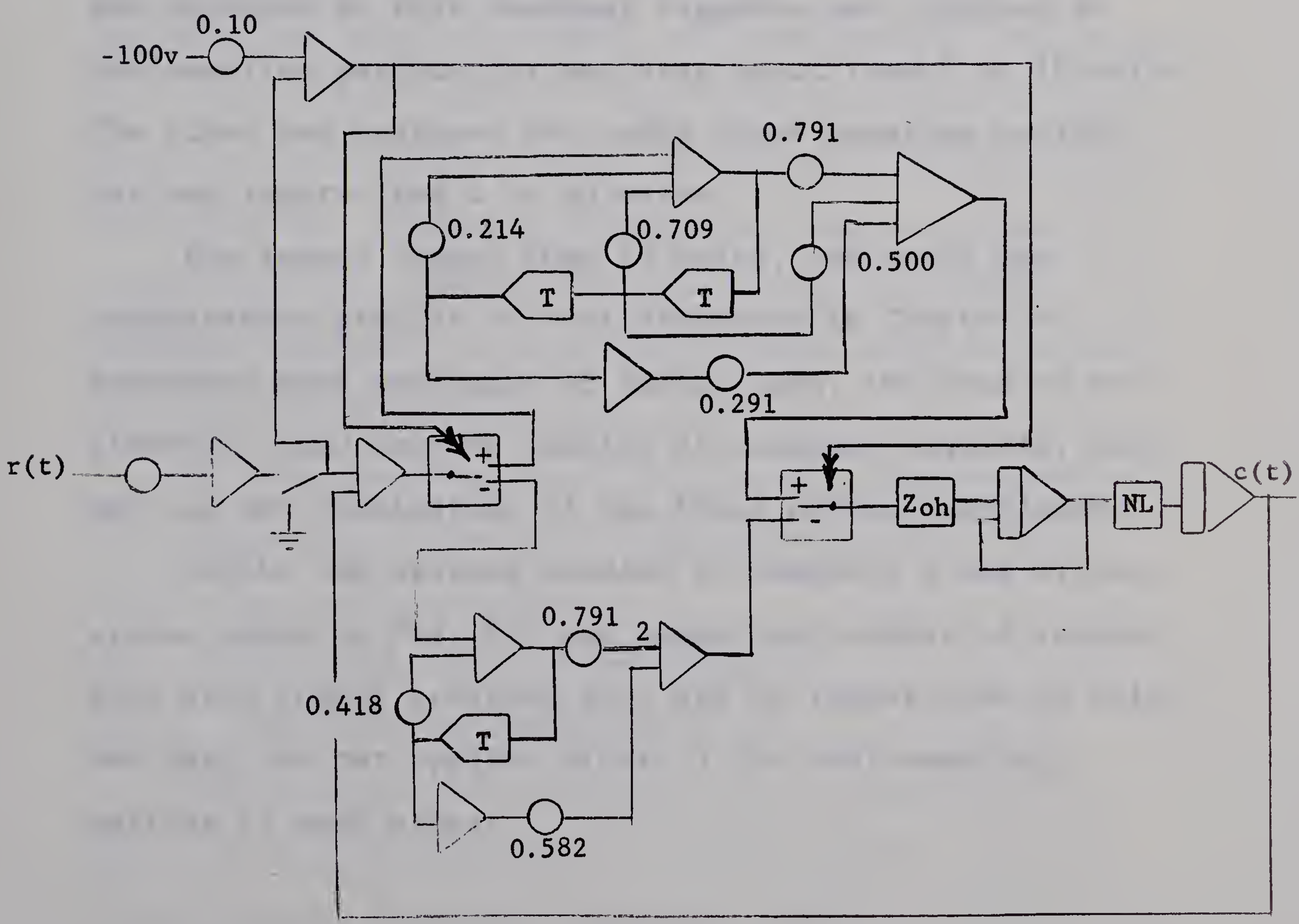


Fig. 6-5

6-3 CONCLUSIONS

In this chapter a different method of feedforward compensation was studied. The feedforward channel was used only to switch comparators which selected one of two digital controllers. The two digital controllers were designed so that the response of the system was minimal as long as the system operated over the linear portion of the nonlinear element. One digital controller was designed so that deadbeat response was obtained in two sampling periods for any step input from 0 to 10 volts. The other was designed for using three sampling periods for any inputs from 0 to 20 volts.

For inputs larger than 20 volts, one could use compensation similar to that discussed in Chapter 4. Depending upon the range of inputs used, the type of non-linearity used and the quality of response required, one may use any combination of the three methods mentioned.

Unlike the systems studied in Chapters 3 and 4, the system shown in Fig. 6-5 can accept any number of successive step inputs provided they are no larger than 20 volts and they are not applied within 3 (or two) sampling periods of each other.

CHAPTER 7

ADDITIONAL MODIFICATIONS USING FEEDFORWARD TECHNIQUES

In previous chapters, the systems which were studied were considered to be initially at rest. The compensation was then studied, assuming that any step input, over a specific range of magnitudes, could be applied. The plant nonlinearity was also assumed to be symmetrical. That is, the gain characteristic was assumed to be identical for positive and negative inputs. No consideration was given as to what would happen if the input was suddenly removed or if successive inputs were applied or if negative inputs were applied. A linear system, using the correct digital controller, will respond in any of these situations with deadbeat response. A nonlinear system, however, in many of these situations will not.

In this chapter, two different nonlinear systems will be compensated so that the response in any of these situations will be deadbeat.

7-1 SYSTEM WITH ISOLATED NONLINEARITY

7-1-A NONSYMMETRICAL NONLINEARITY

With reference to Chapter 3, the system shown in Fig. 7-1 will be considered. In this system, the nonlinearity was symmetrical and as a result, one D.F.G. could be used to generate K_0^f and K_1^f. With a nonsymmetrical

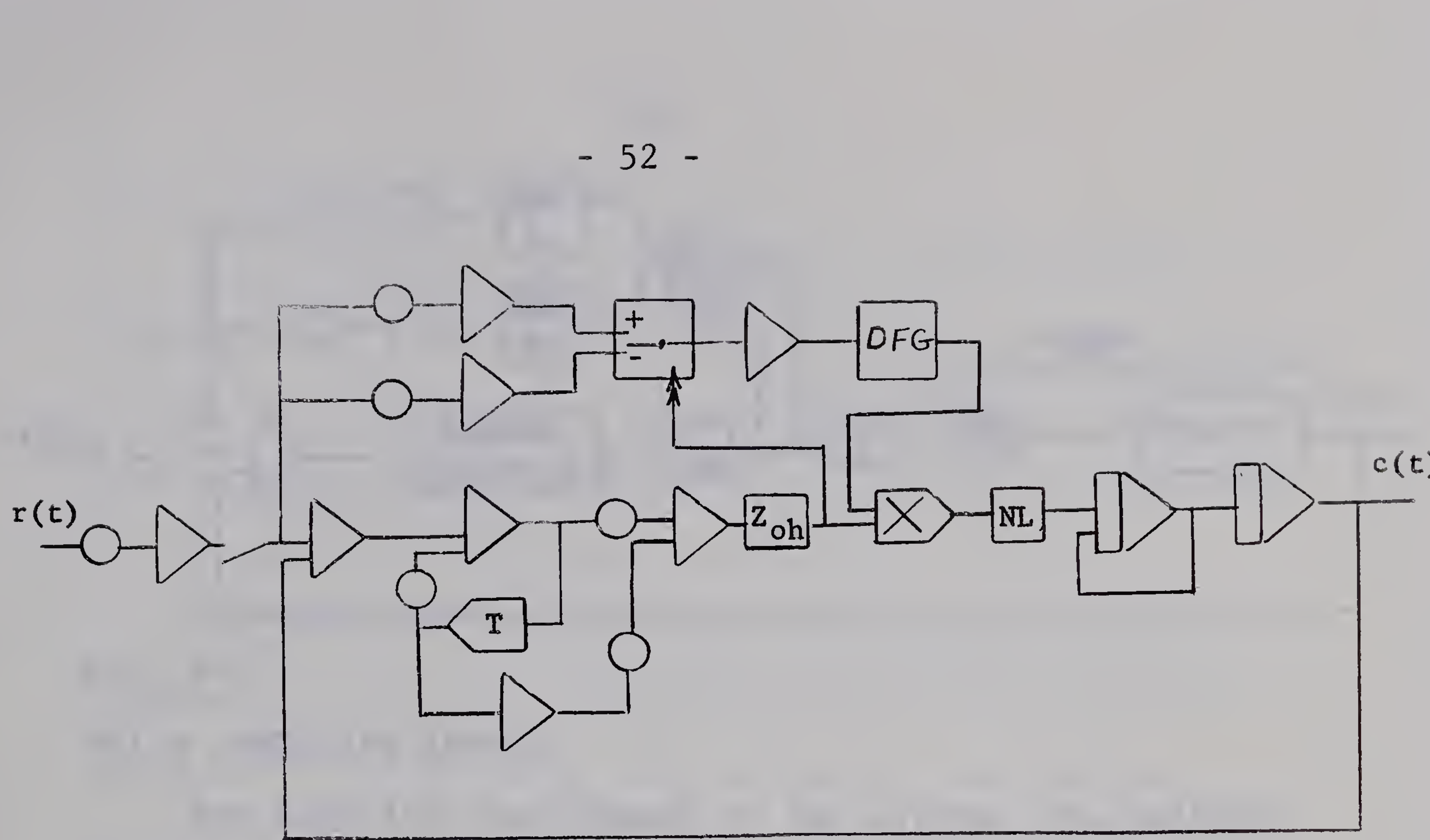


Fig. 7-1

nonlinearity, two separate nonlinear functions must be generated, one determined by the positive portion of the nonlinearity and the other depending upon the negative portion. The feedforward portion of the system would need to be modified as shown in Fig. 7-2. The magnitude scaling has been eliminated for simplification. D.F.G. 'A' is the nonlinear function generator which generates the factor by which the first positive step, $h(0^+)$, is multiplied (assuming a positive step input is applied to the system). Its characteristic is determined by the positive region of the plant nonlinearity. D.F.G. 'B' generates the factor by which the negative step, $h(T^+)$, must be multiplied. The comparator selects the appropriate signal during the correct time interval.

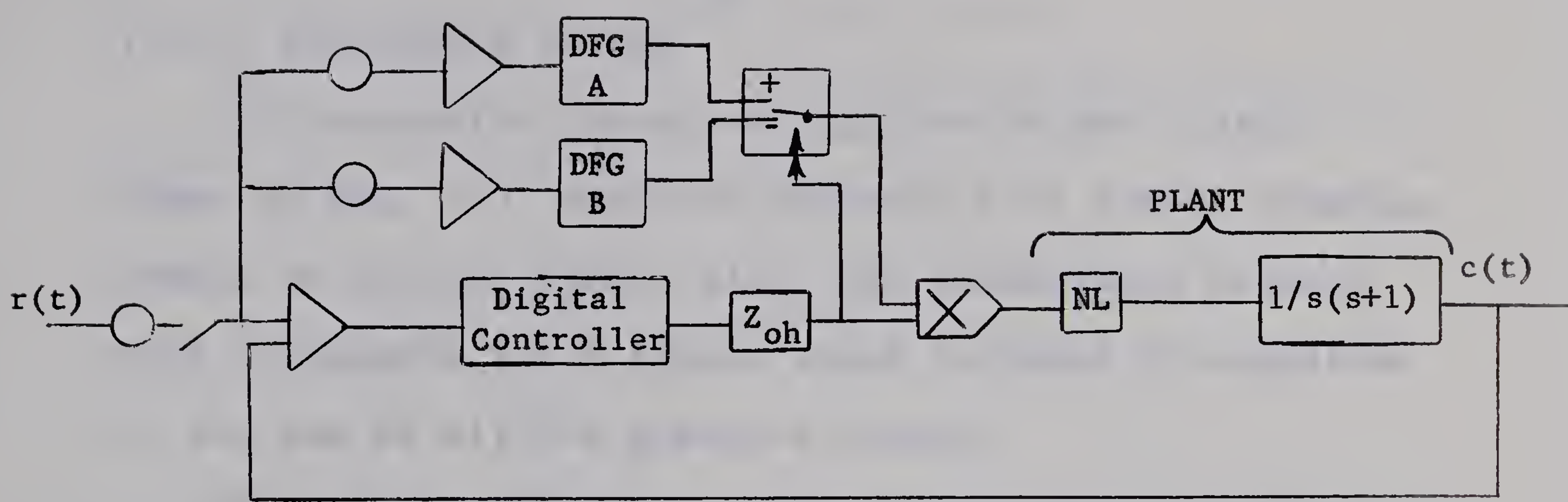


Fig. 7-2

7-1-B NEGATIVE INPUTS

For negative step inputs to the system, the initial input to the plant is a negative step. During the time interval, $T < t < 2T$, a positive step is applied. It can be seen that the same feedforward nonlinear elements (as in Fig. 7-2) can be used for negative inputs as long as the appropriate scaling factors are used. The scale factors required are in the ratio of $|h(0^+)|$ to $|h(T^+)|$. Thus, one D.P.D.T. relay can be used to select the appropriate scaling factors. The circuit can then be modified as shown in Fig. 7-3.

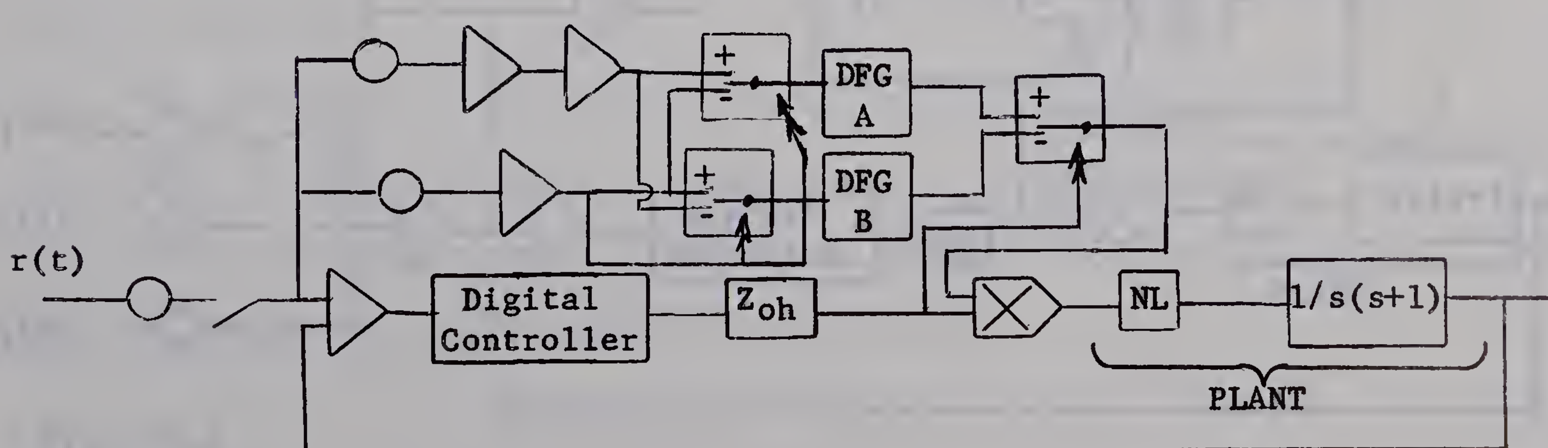


Fig. 7-3

7-1-C SUCCESSIVE INPUTS

If successive inputs are applied to the system shown in Fig. 7-3, deadbeat response will not be obtained except in special cases, since the feedforward network will compensate for a signal which is equal in magnitude to the sum of all the previous inputs.

To overcome this problem, an arrangement must be devised by which the feedforward network can revert to the conditions for zero input after the system has responded to the previous input. If, when the next input is applied, the feedforward network senses only this applied input, Then the system will respond in a deadbeat fashion. The arrangement used to do this uses two time delays (see Fig. 7-4). The system as shown in Fig. 7-4 will also respond with minimal response when any of the inputs is removed.

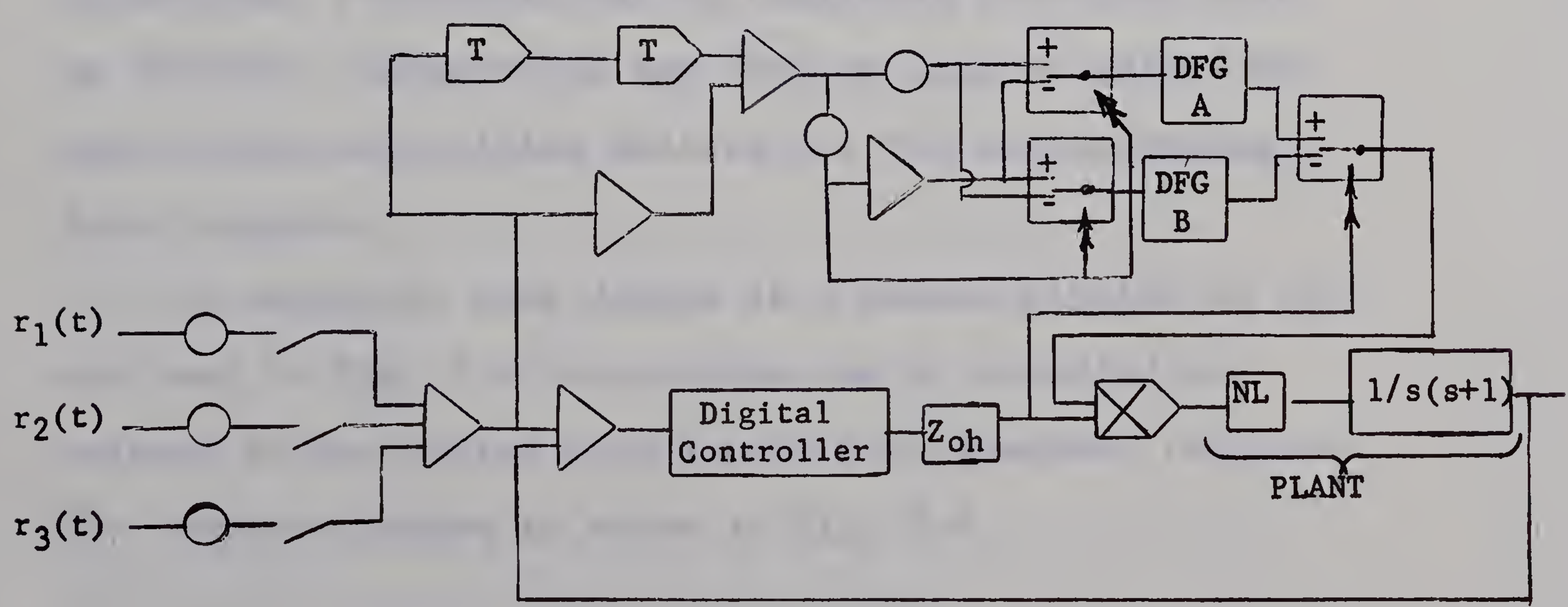


Fig. 7-4

7-2 SYSTEM WITH 'SANDWICHED' NONLINEARITY

With reference to Fig. 4-5, the following simplified system will be considered (Fig. 7-5).

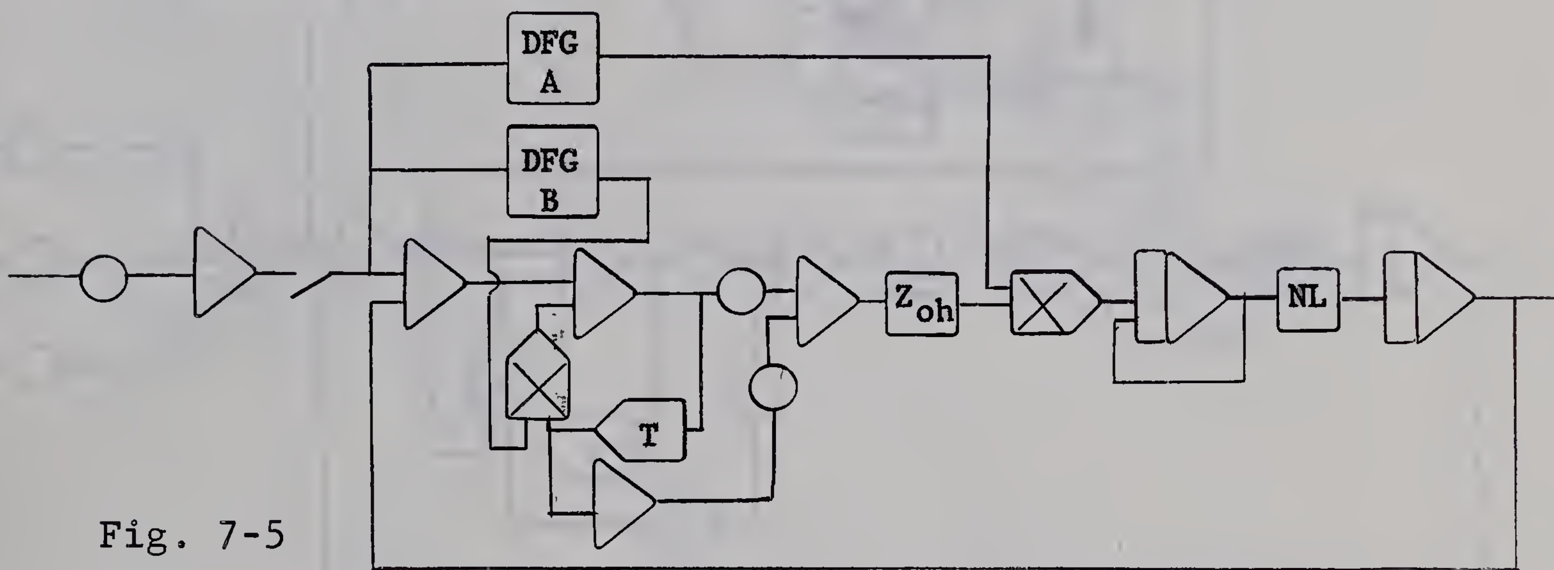


Fig. 7-5

For positive step inputs, the system operates only over the positive portion of the nonlinearity. Thus a non-symmetrical nonlinearity affects the system response only when negative step inputs are applied or when positive inputs are removed. To compensate for these situations, a separate set of nonlinear functions must be derived. Comparators can then be used to select the appropriate multiplying factors for the corresponding input signals.

By using two time delays in a scheme similar to the one used in Fig. 7-4, the system can be modified to respond to successive step inputs with deadbeat response. The complete system is shown in Fig. 7-6.

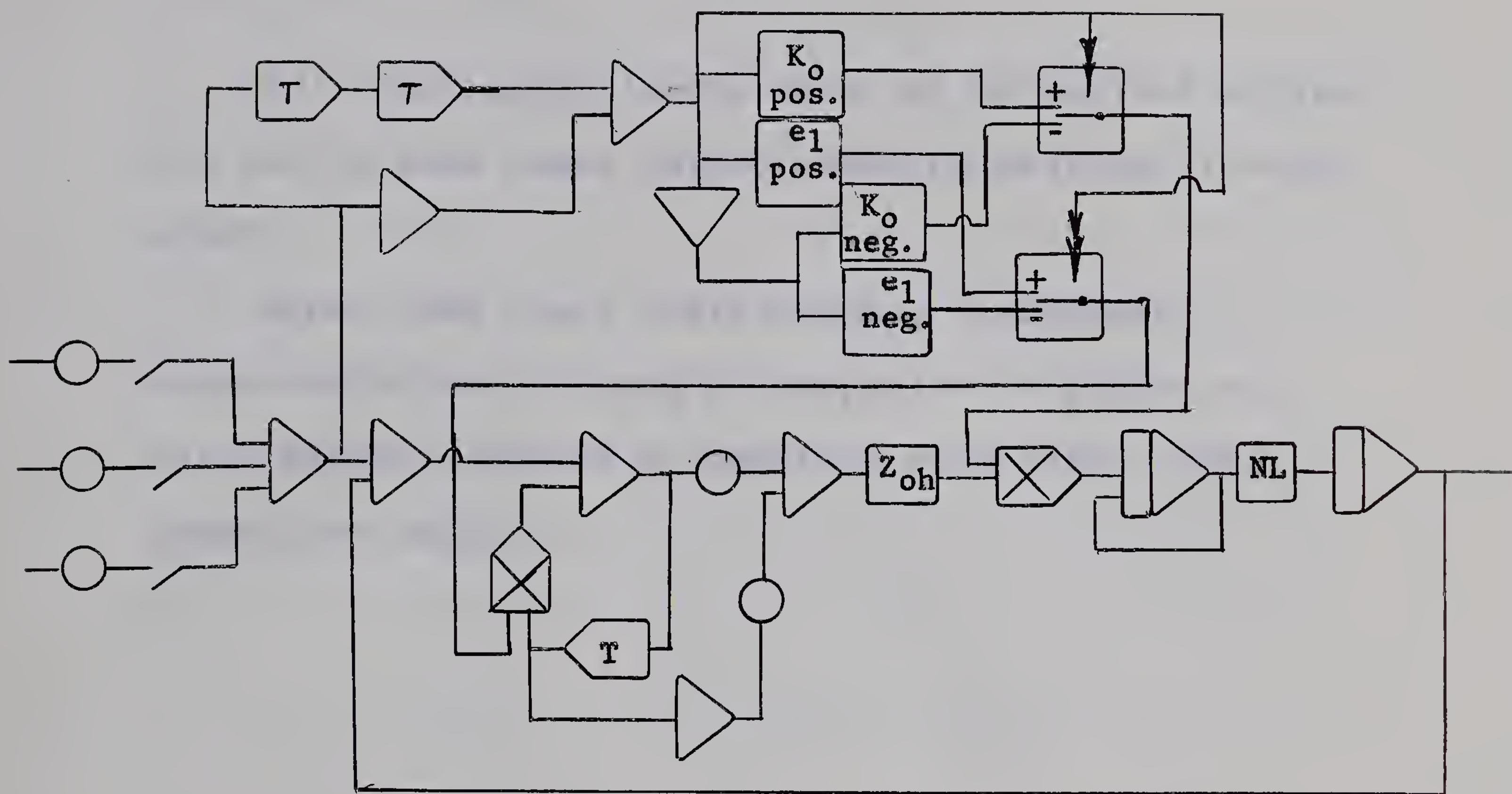


Fig. 7-6

7-3 CONCLUSION

In this chapter, the systems were compensated so that deadbeat response was obtained in many special situations. The compensation used here may or may not be useful depending upon the inputs being considered and the response required. However, using the methods outlined, a nonlinear system can be compensated so that the response is minimal for almost any of a variety of inputs. The only restrictions are that:

(1) The input must be a step input.

(2) The magnitude of the step must be less than the physical limitations of the system.

(3) Successive inputs must not be applied within two (or in some cases three) sampling periods of each other.

Other than these restrictions, feedforward compensation can be used to compensate a system to which either positive or negative successive step inputs are applied.

APPENDIX A

SIMULATION OF THE DIGITAL CONTROLLER

Through most of this thesis, the Digital Controller was simulated using a Digital Controller Box which was recently designed and built by the department especially for this purpose. An explanation of its operation using block diagrams, will be given in this appendix.

The two main requirements in simulating a digital controller are (1) a device for delaying a signal for 1, 2, ... , n sampling periods, if D(z) is of the form:

$$D(z) = \frac{a_o + a_1 z^{-1} + \ldots + a_n z^{-n}}{1 + b_1 z^{-1} + \ldots + b_n z^{-n}} ,$$

and (2) a device to hold an instantaneous signal throughout one sampling period (zero-order hold).

A unit time delay can be simulated by using two zero-order holds. If the second zero-order hold samples and holds the output of the preceeding one, immediately before the preceeding one samples and holds, then the output of the second is equal to the output of the first delayed by one sampling period. This is valid only on the assumption that the input is described by its values at the sampling instants.

By following a unit time delay with an additional zero-order hold which samples an instant before the preceeding one, an additional time delay is generated.

In Fig. A-1 a block diagram schematically outlines the method used in generating these time delays.

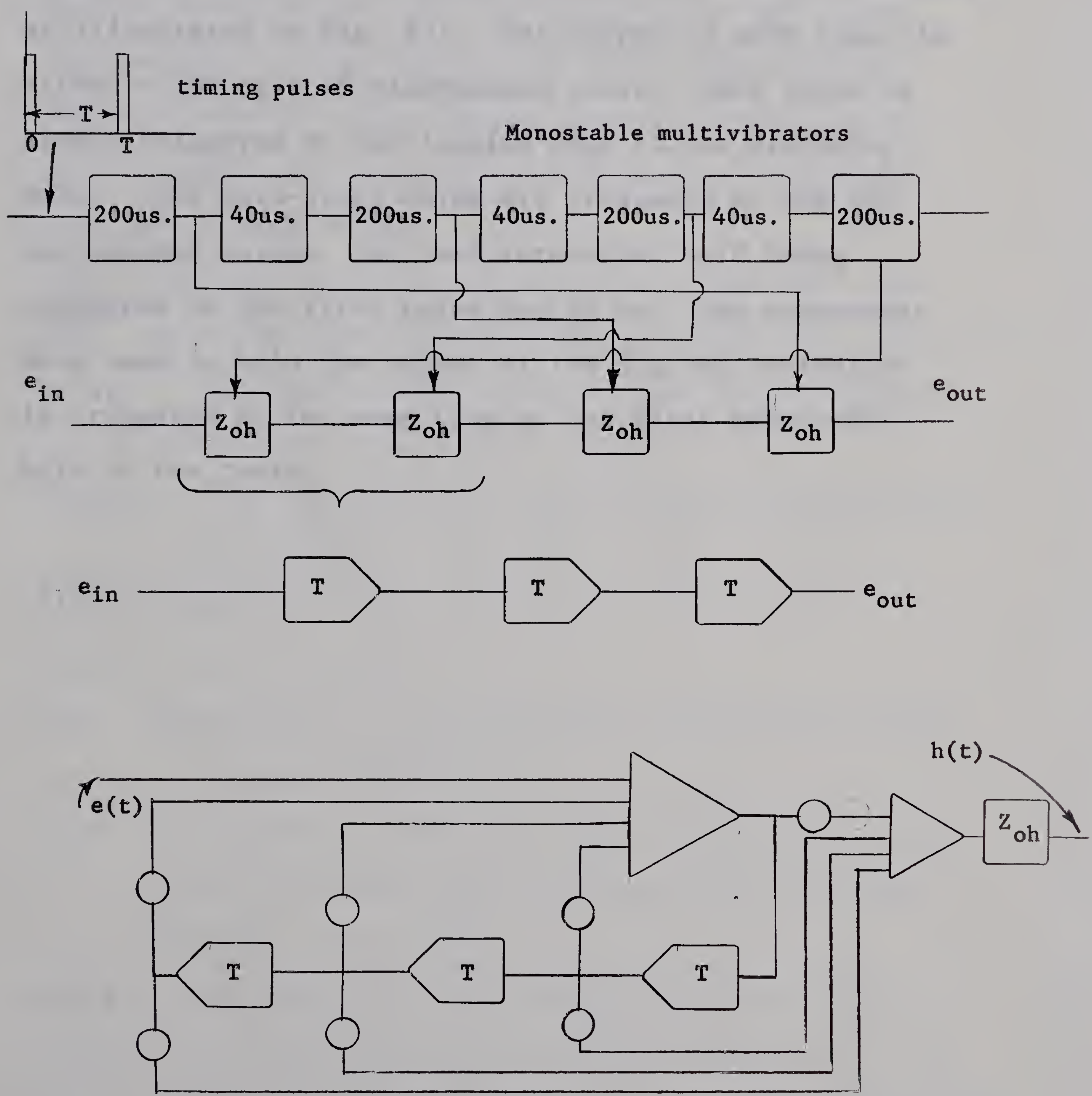


Fig. A-1

Timing pulses, generated by an accurate timing clock, trigger the first of a series of monostable multivibrators as illustrated in Fig. A-1. The output of each stage is either a 200 or a 40 microsecond pulse. Each stage is always triggered by the lagging edge of the preceding pulse. The zero-order holds are triggered by the 200 microsecond pulses, the last zero-order hold being triggered by the first pulse and so on. The zero-order hold used to hold the output of the digital controller is triggered at the same time as the first zero-order hold in the train.

APPENDIX B

DIGITAL COMPUTER PROGRAM

In Chapter 4, a method was outlined wherein the digital computer was used to calculate the points on the nonlinear curves used in the feedforward network. The flow sheet for the computer program as well as the program itself are shown in this appendix. The symbols used are defined below.

A(N) - ordinate break points of the nonlinearity. (See
B(N) - abscissa break points of the nonlinearity. Fig. 4-4)

T(I,J) - time required for $x_2(t)$ to rise to B(J+1) during the test where $x_2(T)$ = B(I+1).

TT(I,J) - Time required for $x_2(t)$ to die down from B(I+1) to B(I-J+1)

S(I,J) = EXP(-T(I,J))

SS(I,J) = EXP(-TT(I,J))

H(I) corresponds to the value $h(0^+)$ required for $x_2(T)$ to equal B(I+1).

HH(I) corresponds to the value $h(T^+)$ required for $x_2(t)$ to die down from B(I+1) to zero during the time interval, $T < t < 2T$.

ARA(I) - area under the curve of $x_2^*(t)$ between $0 < t < T$.

ARB(I) - area under the curve of $x_2^*(t)$ between $T < t < 2T$.

SL(I) - slope of the line segment of the plant nonlinearity between A(I),B(I) and A(I+1),B(I+1).

C(I) - abscissa intercept of the line segment with slope SL(I) as described above.

N corresponds to the number of line segments making up the plant nonlinearity.

AK = K_o

BK = K_1

RO = R_o

FAK = K_o^f = Feedforward multiplying constant during the first sampling period.

FBK = K_1^f

ER = $e(T^+)$

The flow diagram for the computer program is shown in Fig. B-1, the program itself being shown in Fig. B-2.

DIGITAL COMPUTER FLOW CHART

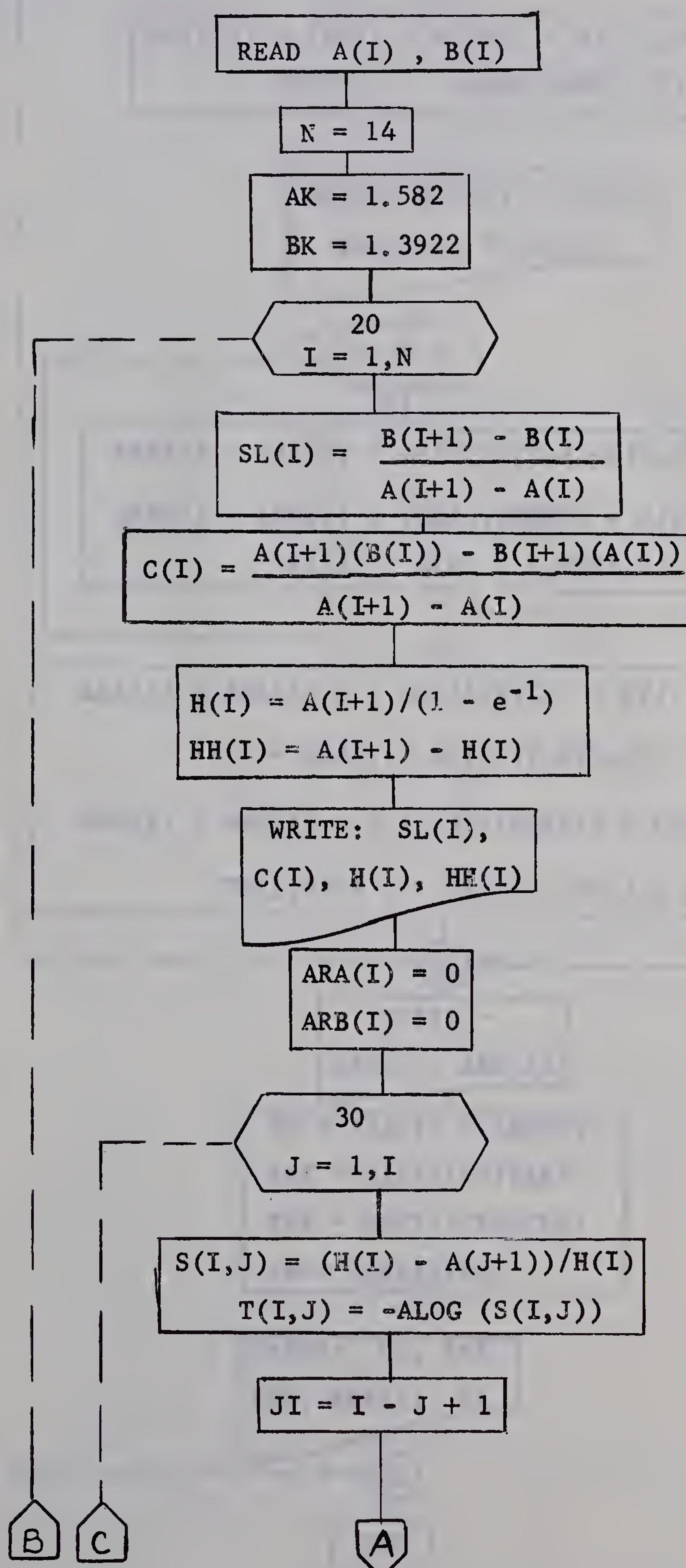
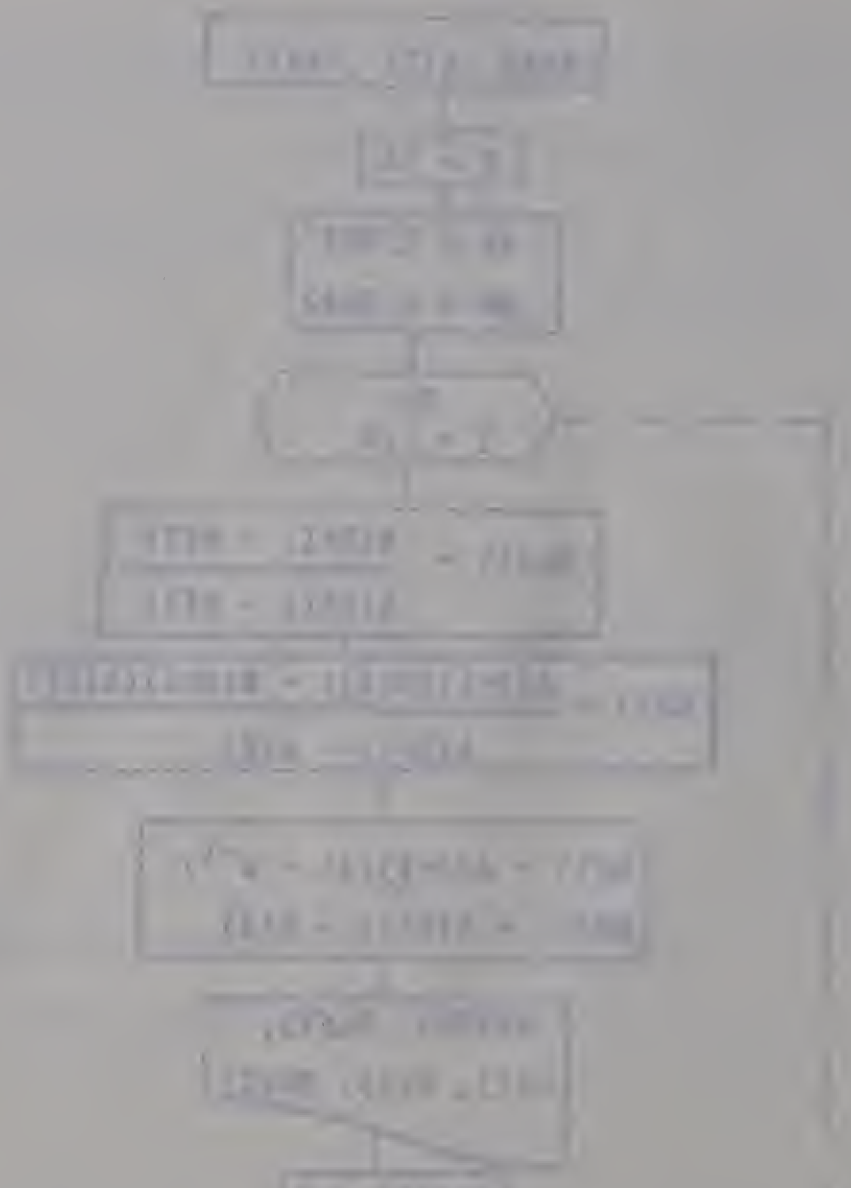


Fig. B-1



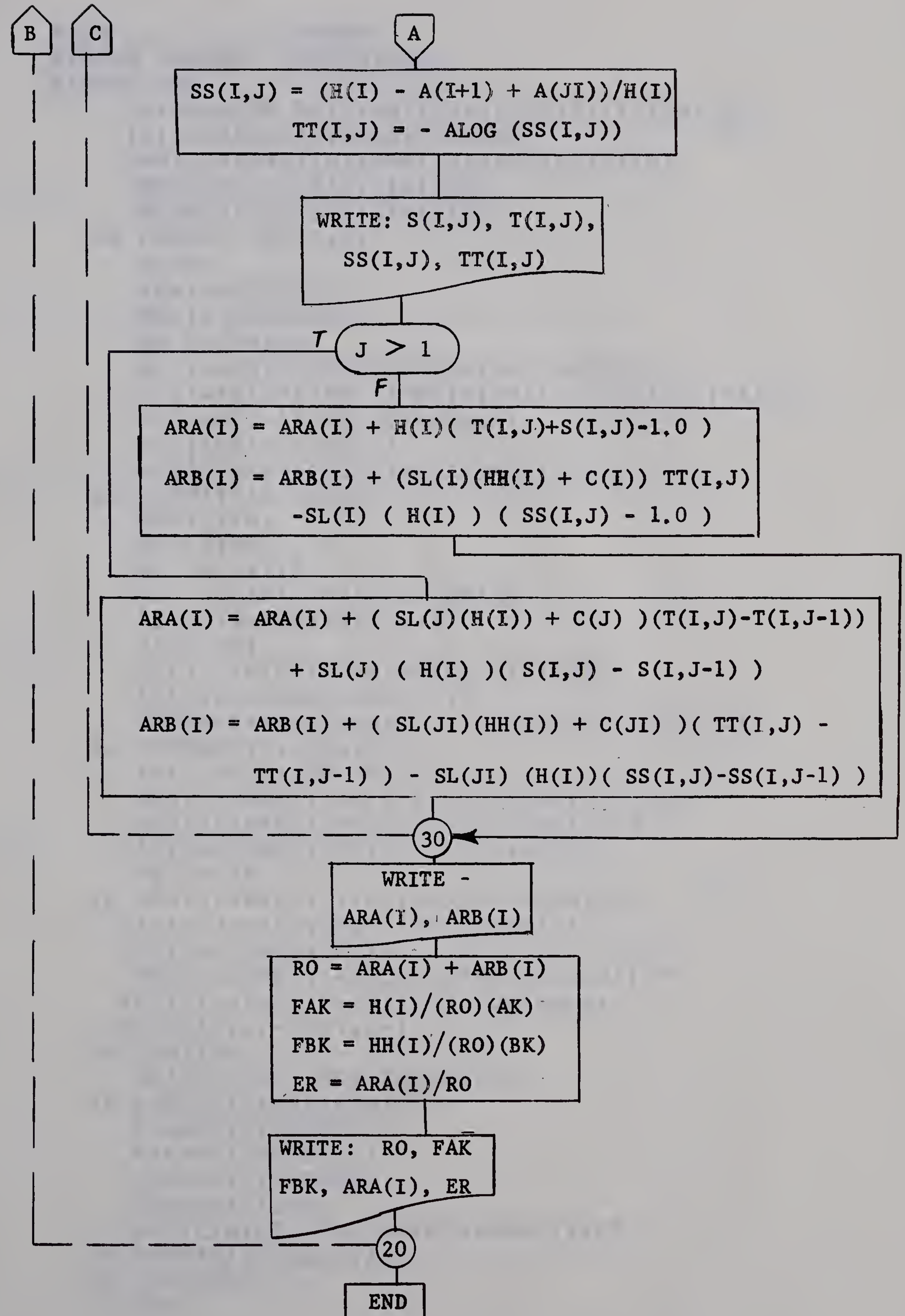

B
C
A
SS(I,J) = (H(I) - A(I+1) + A(JI))/H(I)
TT(I,J) = - ALOG (SS(I,J))
WRITE: S(I,J), T(I,J), SS(I,J), TT(I,J)
T
J > 1
F
ARA(I) = ARA(I) + H(I)(T(I,J)+S(I,J)-1.0)
ARB(I) = ARB(I) + (SL(I)(HH(I) + C(I)) TT(I,J) -SL(I) (H(I)) (SS(I,J) - 1.0)
ARA(I) = ARA(I) + (SL(J)(H(I)) + C(J))(T(I,J)-T(I,J-1)) + SL(J) (H(I))(S(I,J) - S(I,J-1))
ARB(I) = ARB(I) + (SL(JI)(HH(I)) + C(JI))(TT(I,J) - TT(I,J-1)) - SL(JI) (H(I))(SS(I,J)-SS(I,J-1))
30
WRITE - ARA(I), ARB(I)
RO = ARA(I) + ARB(I)
FAK = H(I)/(RO)(AK)
FBK = HH(I)/(RO)(BK)
ER = ARA(I)/RO
WRITE: RO, FAK FBK, ARA(I), ER
20
END

Fig. B-1 (cont)

```
$JOB               345009
$IBJOB FORCOM  NODECK,NOGO
$IBFTC GRK
      DIMENSION A(15),B(15),T(15,15)(TT(15,15),
     1S(15,15),SS(15,15),H(14),
     2HH(15),ARA(14),ARB(14),SL(14),C(14)
      READ(5,10)(A(I),I=1,15)
      READ(5,10)(B(I),I=1,15)
   10 FORMAT (6E12.5)
      N=14
      AK=1.58197671
      BK=1.39221119
      DO 20 I=1,N
      SL(I)=(B(I+1)-B(I))/(A(I+1)-A(I))
      C(I)=(A(I+1)*B(I)-B(I+1)*A(I))/(A(I+1)-A(I))
      H(I)=A(I+1)/(1.-EXP(-1.))
      HH(I)=A(I+1)-H(I)
      WRITE(6,12) SL(I),C(I),H(I),HH(I)
   12 FORMAT(1X,4E16.6)
      ARA(I)=0.
      ARB(I)=0.
      DO 30 J=1,I
      S(I,J)=(H(I)-A(J+1))/H(I)
      T(I,J)=-ALOG(S(I,J))
      JI=I-J+1
      SS(I,J)=(H(I)-A(I+1)+A(JI))/H(I)
      TT(I,J)=-ALOG(SS(I,J))
      WRITE(6,14)S(I,J),T(I,J),SS(I,J),TT(I,J)
   14 FORMAT(1X,4E16.6)
      IF(J.GT.1) GO TO 15
      ARA(I)=ARA(I)+H(I)*(T(I,J)+S(I,J)-1.)
      ARB(I)=ARB(I)+(SL(I)*HH(I)+C(I))*
     1TT(I,J)-SL(I)*H(I)*(SS(I,J)-1.)
      GO TO 30
   15 ARA(I)=ARA(I)+(SL(J)*H(I)+C(J))*
     1(T(I,J)-T(I,J-1))+SL(J)*H(I)*
     2(S(I,J)-S(I,J-1))
      ARB(I)=ARB(I)+(SL(JI)*HH(I)+C(JI))*
     1(TT(I,J)-TT(I,J-1))-SL(JI)*H(I)
     2*(SS(I,J)-SS(I,J-1))
   30 CONTINUE
      WRITE(6,35) ARA(I),ARB(I)
   35 FORMAT(1X,2E16.6)
      RO=ARA(I)+ARB(I)
      FAK=H(I)/(RO*AK)
      FBK=HH(I)/(RO*BK)
      ER=ARA(I)/RO
      WRITE(6,25) RO,FAK,FBK,ARA(I),ER
   25 FORMAT(1X,5E13.5)
   20 CONTINUE
      END
```

Fig. B-2

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